Machine Learning Notes 11

Random Forrest Algorithm

Information Gain

- Slides from https://www.cs.cmu.edu
- Information gain is one criteria to decide on the
- attribute.

Information

- Imagine:
- 1. Someone is about to tell you your own name
- 2. You are about to observe the outcome of a dice roll
- 2. You are about to observe the outcome of a coin flip
- 3. You are about to observe the outcome of a biased coin flip

- Each situation have a different *amount of uncertainty*
- as to what outcome you will observe.

Information

- Information:
- reduction in uncertainty (amount of surprise in the outcome)

$$I(E) = \log_2 \frac{1}{p(x)} = -\log_2 p(x)$$

If the probability of this event happening is small and it happens the information is large.

- Observing the outcome of a coin flip is head ⟶ $I = -\log_2 1/2 = 1$

2. Observe the outcome of a dice is 6 ⟶ $I = -\log_2 1/6 = 2.58$

Entropy

- The *expected amount of information* when observing the output of a random variable X

$$H(X) = E(I(X)) = \sum_i p(x_i) I(x_i) = -\sum_i p(x_i) \log_2 p(x_i)$$

If there X can have 8 outcomes and all are equally likely

$$H(X) == -\sum_i 1/8 \log_2 1/8 = 3 \quad \text{bits}$$

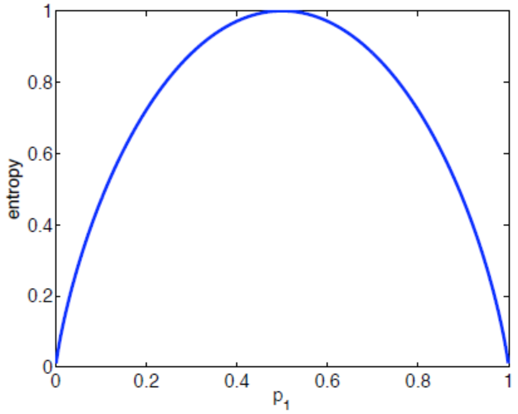
Entropy

- If there are *k* possible outcomes

$$H(X) \leq \log_2 k$$

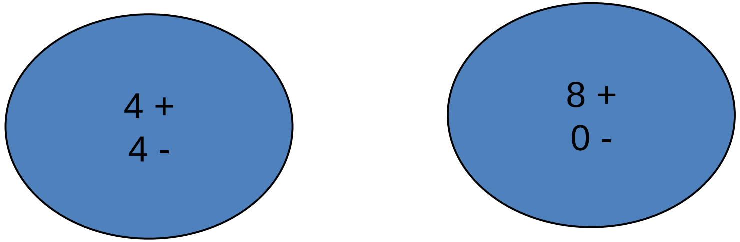
- Equality holds when all outcomes are equally likely

- The more the probability dis
- the deviates from
- uniformity
- the lower the entropy



Entropy, purity

- Entropy measures the purity



The distribution is less uniform
Entropy is lower
The node is purer

Conditional entropy

$$H(X) = -\sum_i p(x_i) \log_2 p(x_i)$$

$$H(X|Y) = -\sum_j p(y_j) H(X|Y = y_j)$$

$$= -\sum_j p(y_j) \sum_i p(x_i|y_j) \log_2 p(x_i|y_j)$$

Information gain

- $IG(X,Y)=H(X)-H(X|Y)$

Reduction in uncertainty by knowing Y

Information gain:
(information before split) – (information after split)

Information Gain

- Information gain:
- (information before split) – (information after split)

Example

Attributes Labels

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

Which one do we choose X1 or X2?

IG(X1,Y) = H(Y) – H(Y|X1)

H(Y) = - (5/10) log(5/10) -5/10log(5/10) = 1

H(Y|X1) = P(X1=T)H(Y|X1=T) + P(X1=F) H(Y|X1=F)

= 4/10 (1log 1 + 0 log 0) +6/10 (5/6log 5/6 +1/6log1/6)

= 0.39

Information gain (X1,Y)= 1-0.39=0.61

Which one do we choose?

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

Information gain (X1,Y)= 0.61

Information gain (X2,Y)= 0.12

Pick the variable which provides the most information gain about Y

Pick X1

Recurse on branches

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

One branch

The other branch

Caveats

- The number of possible values influences the information gain.

- The more possible values, the higher the gain (the more likely it is to form small, but pure partitions)

Purity (diversity) measures

- Purity (Diversity) Measures:
- – Gini (population diversity)
- – Information Gain
- – Chi-square Test

Overfitting

- You can perfectly fit to any training data
- Zero bias, high variance

- Two approaches:
- 1. Stop growing the tree when further splitting the data does not yield an improvement
- 2. Grow a full tree, then prune the tree, by eliminating nodes.

Bagging

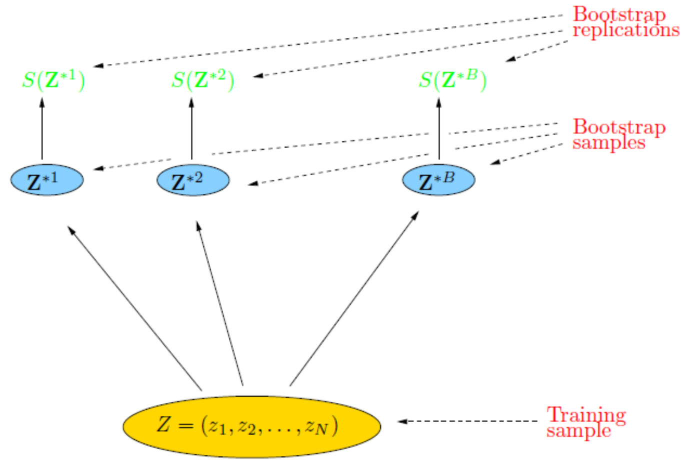
- Bagging or *bootstrap aggregation* a technique for reducing the variance of an estimated prediction function.

- For classification, a *committee* of trees each
- cast a vote for the predicted class.

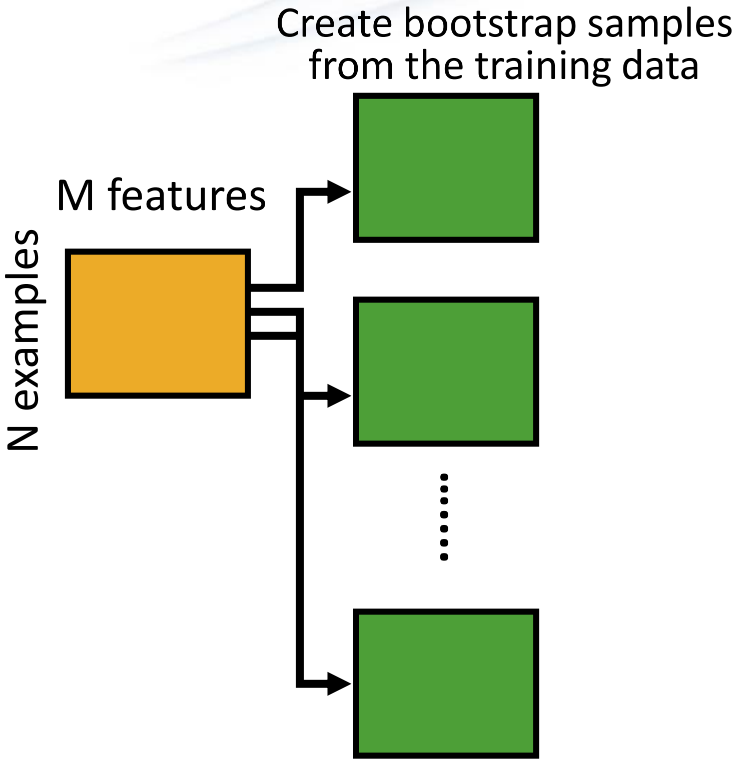
Bootstrap

The basic idea:

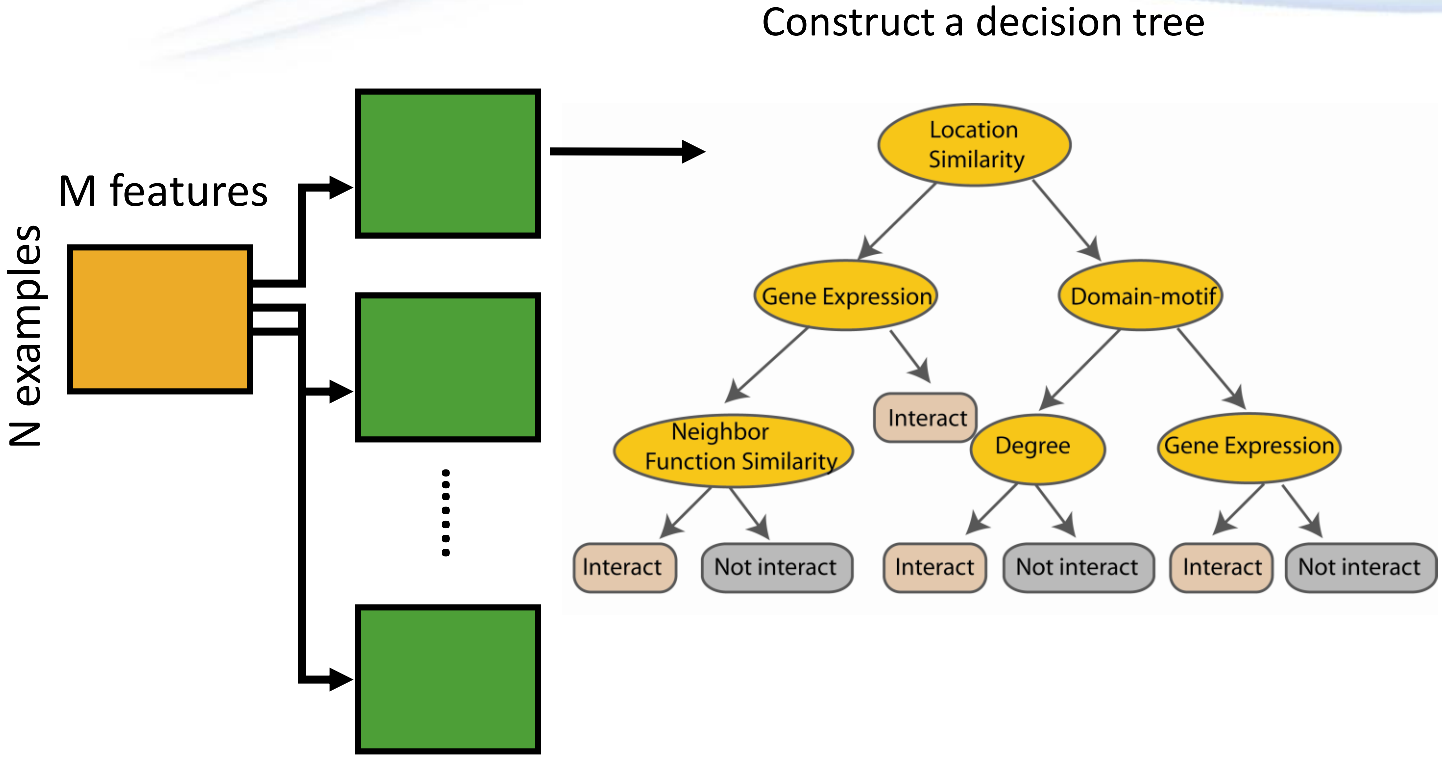
randomly draw datasets *with replacement* from the training data, each sample *the same size as the original training set*



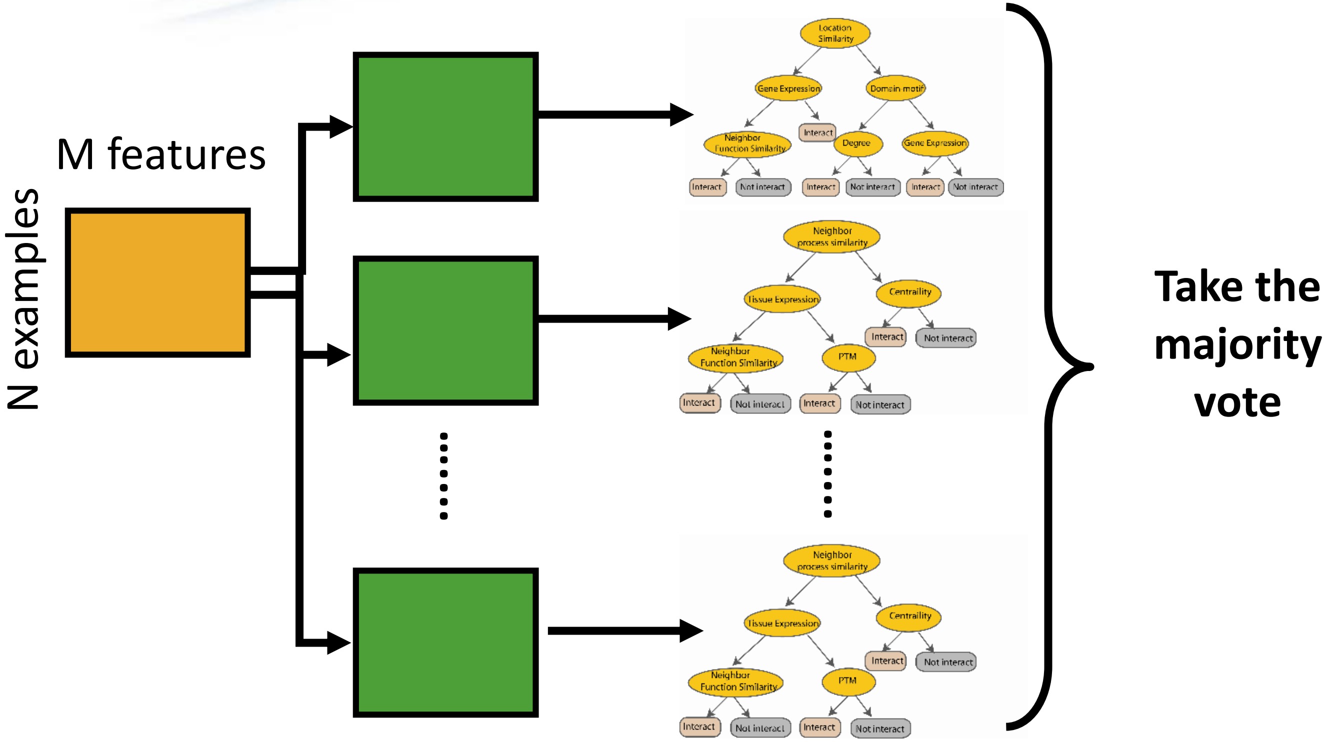
Bagging


Create bootstrap samples
from the training data
M features
N examples

Random Forest Classifier



Random Forest Classifier



Bagging

Z = {(x_1, y_1), (x_2, y_2), . . . , (x_N, y_N)}

Z^{*b} where = 1,.., B..

The prediction at input x when bootstrap sample *b* is used for training

$$\hat{f}_{\text{bag}}(x) = \frac{1}{B}\sum_{b=1}^{B}\hat{f}^{*b}(x).$$

http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf (Chapter 8.7)

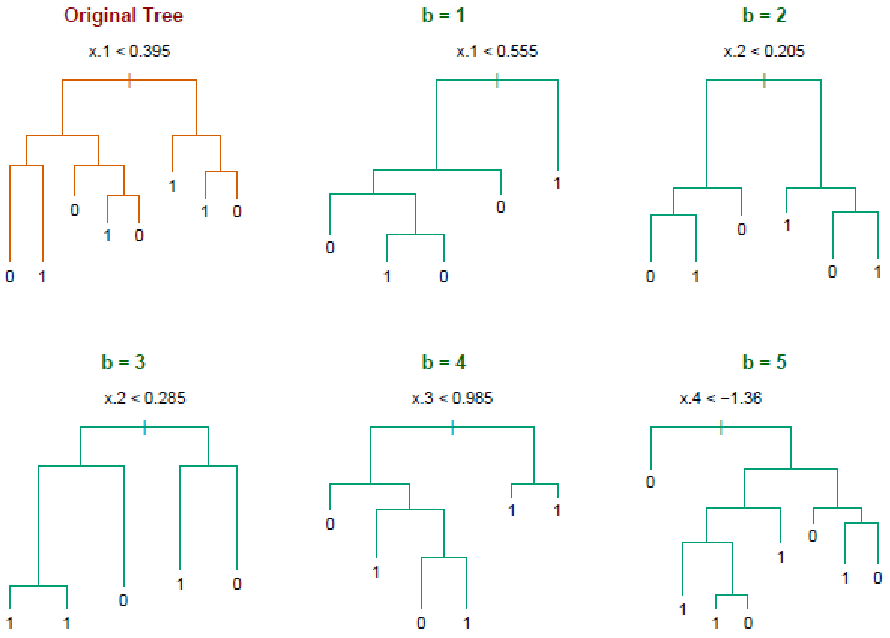
Bagging : an simulated example

- Generated a sample of size $N = 30$, with two
- classes and $p = 5$ features, each having a
- standard Gaussian distribution with pairwise
- Correlation 0.95.

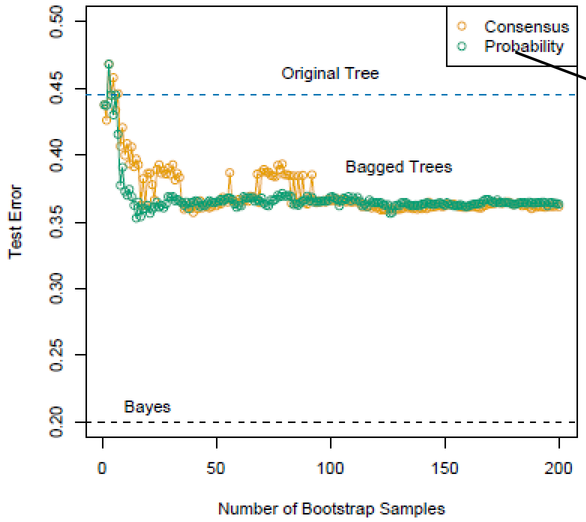
- The response Y was generated according to
- $\Pr(Y = 1/x1 \leq 0.5) = 0.2$,
- $\Pr(Y = 0/x1 > 0.5) = 0.8$.

Bagging

Notice the bootstrap trees are different than the original tree



Bagging



Treat the voting Proportions as probabilities

FIGURE 8.10. *Error curves for the bagging example of Figure 8.9. Shown is the test error of the original tree and bagged trees as a function of the number of bootstrap samples. The orange points correspond to the consensus vote, while the green points average the probabilities.*

bagging helps under squared-error loss, in short because averaging reduces

Hastie

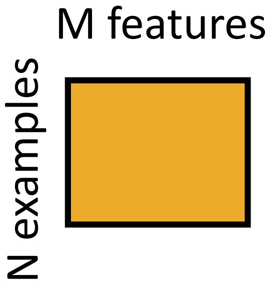
http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf Example 8.7.1

Random forest classifier

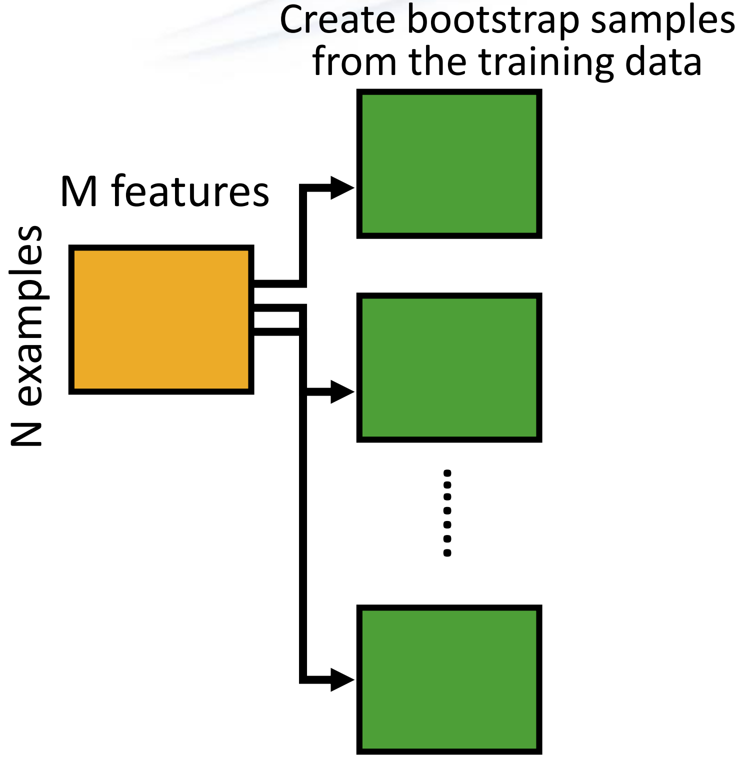
- Random forest classifier, an extension to
- bagging which uses *de-correlated* trees.

Random Forest Classifier

Training Data

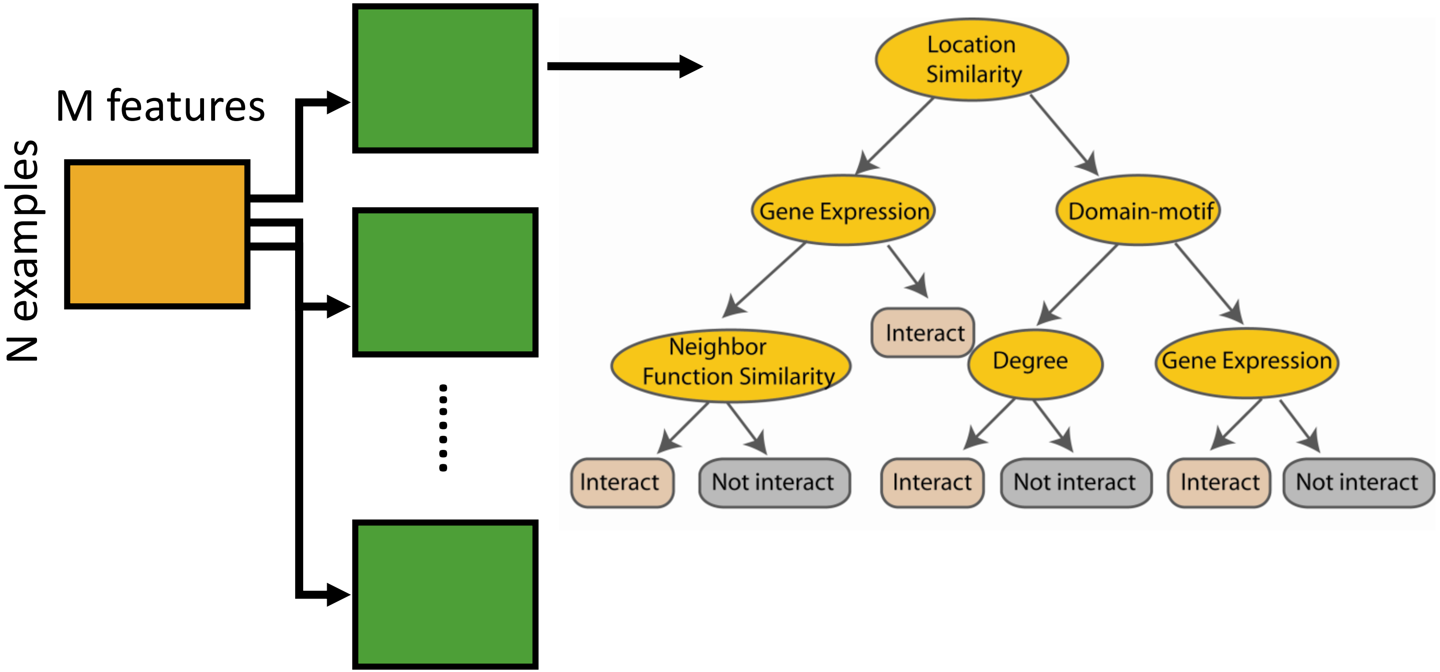


Random Forest Classifier



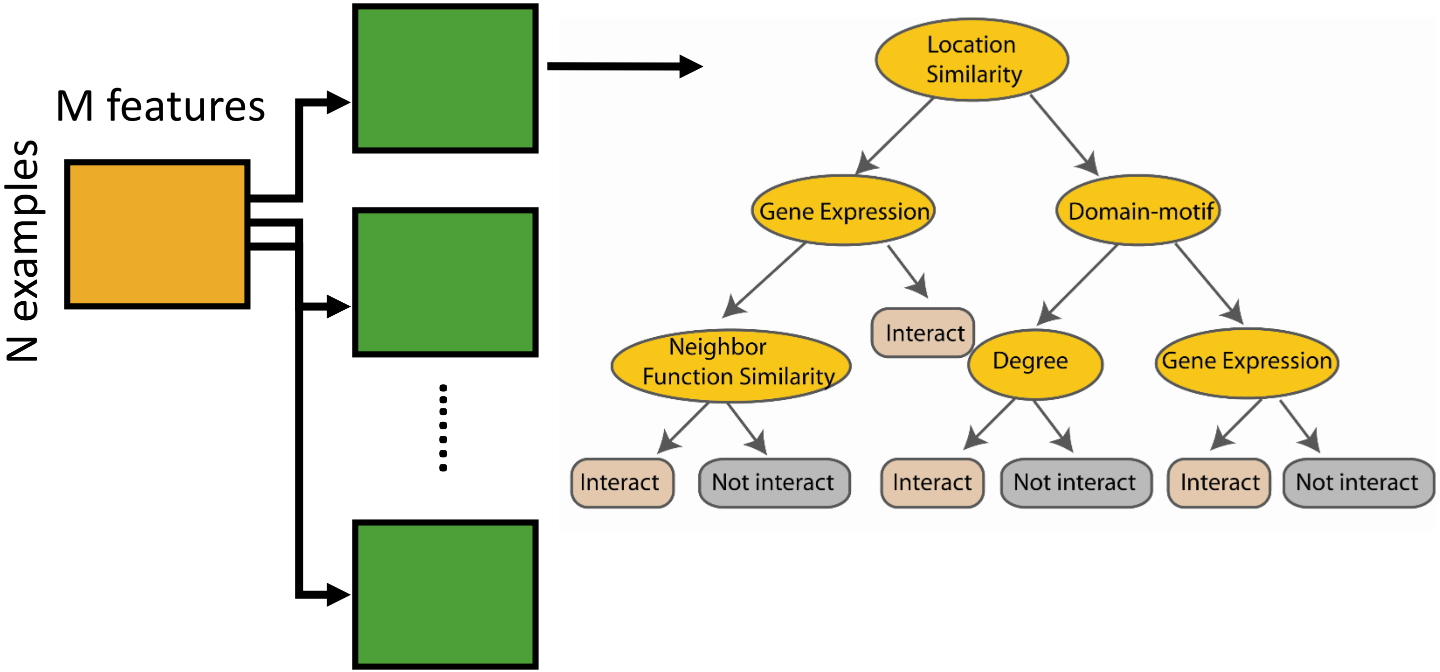
Random Forest Classifier

Construct a decision tree

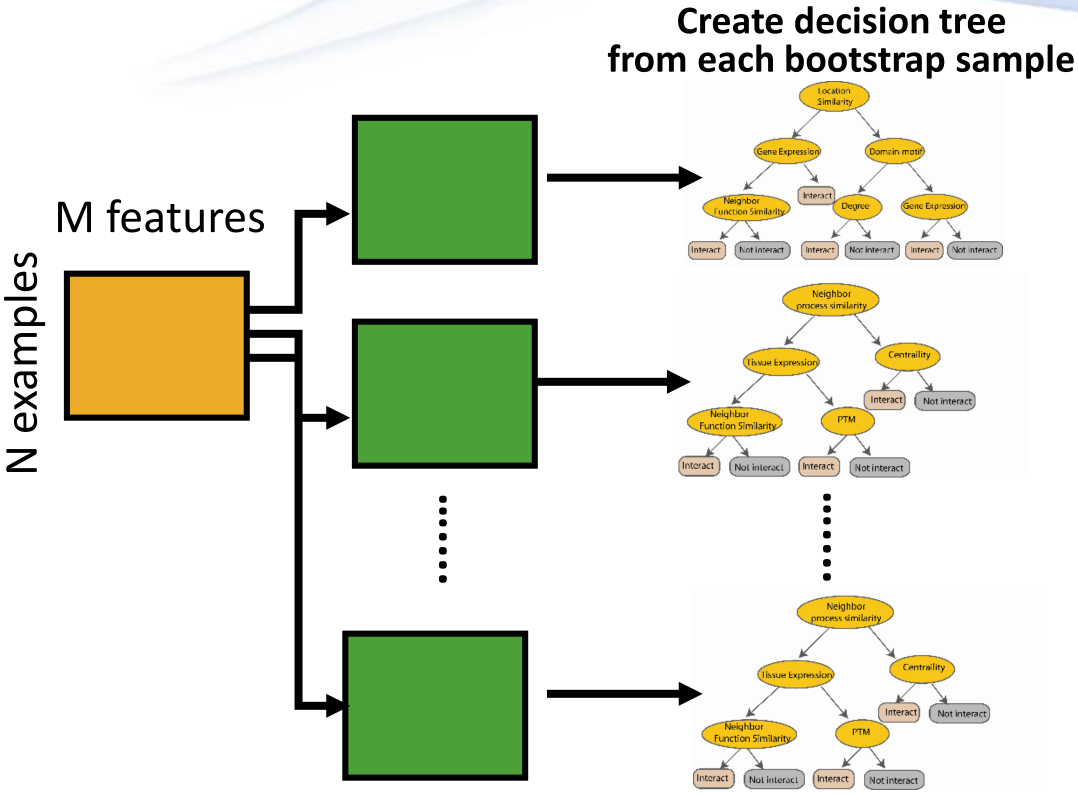


Random Forest Classifier

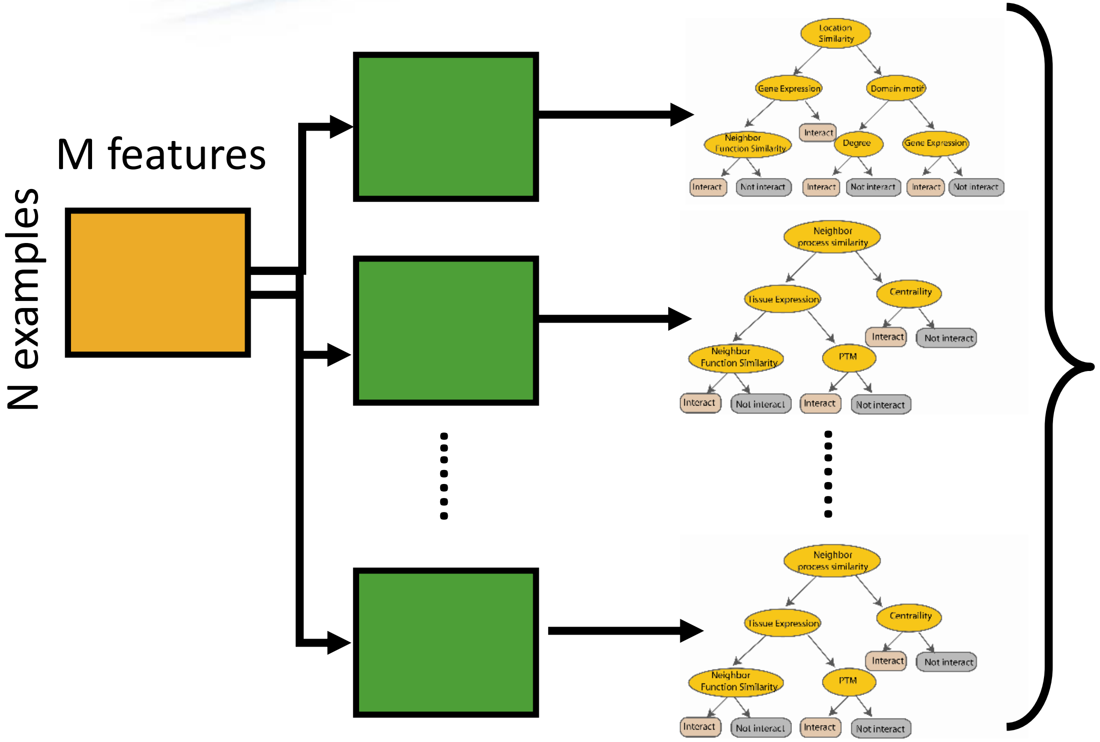
At each node in choosing the split feature choose only among *m*<*M* features



Random Forest Classifier



Random Forest Classifier



Take he majority vote

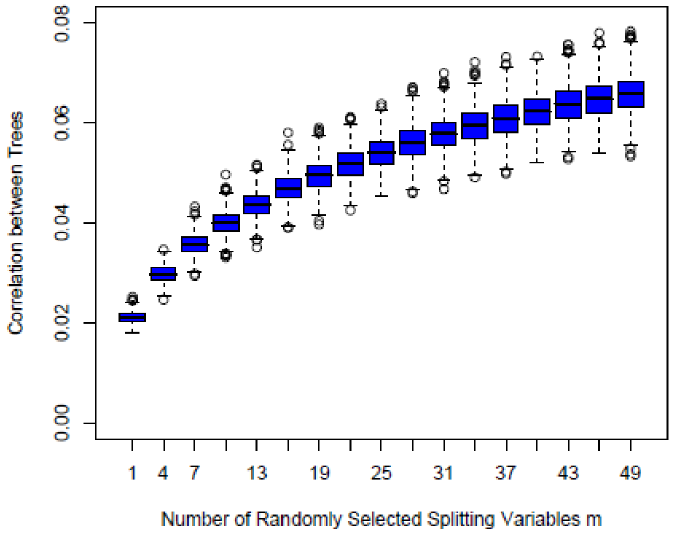


FIGURE 15.9. *Correlations between pairs of trees drawn by a random-forest regression algorithm, as a function of m. The boxplots represent the correlations at 600 randomly chosen prediction points x.*

Random forest

- Available package:
- http://www.stat.berkeley.edu/~breiman/RandomForests/cc_home.htm

- To read more:
- http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf