Faculty of Business Administration

MAT102-Mathematics II / Spring 2020

Exercise-1: Derivatives with Applications

1. Find the derivative of each function.

a) $y = 4$ **b)** $y = 2 + 5t - 8t^3$ **c)** $y = \dfrac{5x^3}{4} - \dfrac{2}{5x^3}$ **d)** $f(x) = \dfrac{5}{9x^7} + 5\sqrt[3]{x}$ **e)** $h(t) = \dfrac{3}{t^{3/5}} - \dfrac{8}{t^{3/2}}$

f) $y = \dfrac{3x-4}{12x^2}$ **g)** $y = \dfrac{t^2 - t}{t^3 + 1}$ **i)** $f(x) = (x^3 + 2x^2)(x^2 - 3x)$ **j)** $y = \dfrac{2x-1}{(x^3+4)(x^2-8)}$ **k)** $y = \dfrac{2\sqrt{x}}{x^2 - 3x + 1}$

2. Let $f(x) = 3x^4 - 6x^2 - 7$.

a) Find $f'(x)$.

b) Find the slope of the the graph of f at $x = 2$.

c) Find the equation of the tangent line at $x = 2$.

d) Find the value(s) of x where the tangent line is horizontal.

3. A company manufactures automatic transmissions for cars. The total weekly cost (in dollars) of producing x transmissions is given by

$$C(x) = 50000 + 600x - 0.75x^2.$$

a) Find the marginal cost function.

b) Find the marginal cost at a production level of 200 transmissions per week and interpret the result.

c) Find the exact cost of producing the 201st transmission.

4. The total cost of producing x bicycles is given by the cost function

$$C(x) = 10000 + 150x - 0.2x^2.$$

a) Find the exact cost of producing the 141st bicycle.

b) Use marginal cost to approximate the cost of producing the 141st bicycle.

5. The total cost (in dollars) of manufacturing x auto body frames is

$$C(x) = 6000 + 300x.$$

a) Find the average cost per unit if 500 frames are produced.

b) Find the marginal average cost at a production level of 500 units and interpret the results.

c) Use the results from parts (a) and (b) to estimate the average cost per frame if 501 frames are produced.

6. The total profit (in dollars) from the sale of x calendars is

$$P(x) = 22x - 0.2x^2 - 400 \qquad 0 \le x \le 100.$$

a) Find the exact profit from the sale of the 41st calendar.

b) Use the marginal profit to approximate the profit from the sale of the 41st calendar.

7. The total profit (in dollars) from the sale of x gas grills is

$$P(x) = 20x - 0.02x^2 - 320 \qquad 0 \le x \le 1000.$$

a) Find the average profit per grill if 40 grills are produced.

b) Find the marginal average profit at a production level of 40 grills and interpret the results.

c) Use the results from parts (a) and (b) to estimate the average profit per grill if 41 grills are produced.

8. The price p (in dollars) and the demand x for a brand of running shoes are related by the equation

$$x = 4000 - 40p.$$

a) Express the price p in terms of the demand $x,$ and find the domain of this function.

b) Find the revenue $R(x)$ from the sale of x pairs of running shoes. What is the domain of R?

c) Find the marginal revenue at a production level of 1600 pairs and interpret the results.

d) Find the marginal revenue at a production level of 2500 pairs and interpret the results.

9. The price–demand equation and the cost function for the production of HDTVs are given, respectively, by

$$x = 9000 - 30p \quad \text{and} \quad C(x) = 150000 + 30x$$

where x is the number of HDTVs that can be sold at a price of $\$p$ per TV and $C(x)$ is the total cost (in dollars) of producing x TVs.

a) Express the price p in terms of the demand $x,$ and find the domain of this function.

b) Find the marginal cost.

c) Find the revenue function and state its domain.

d) Find the marginal revenue.

e) Find $R'(3000)$ and $R'(6000)$ and interpret these quantities.

f) Graph the cost function and the revenue function on the same coordinate system for $0 \le x \le 9000.$ Find the break-even points and indicate regions of loss and profit.

g) Find the profit function in terms of $x.$

h) Find the marginal profit.

I) Find $P'(1500)$ and $P'(4500)$ and interpret these quantities.

10. The total cost and the total revenue (in dollars) for the production and sale of x ski jackets are given, respectively, by

$$C(x) = 24x + 21900 \quad \text{and} \quad R(x) = 200x - 0.2x^2 \quad 0 \le x \le 1000.$$

a) Find the value of x where the graph of $R(x)$ has a horizontal tangent line.

b) Find the profit function $P(x).$

c) Find the value of x where the graph of $P(x)$ has a horizontal tangent line.

d) Graph the $C(x)$, $R(x)$ and $P(x)$ on the same coordinate system for $0 \le x \le 1000$. Find the break-even points. Find the x intercepts of the graph of $P(x)$.