

**Faculty of Business Administration**  
**MAT102-Mathematics II / 2020 Spring**

**Exercise-2 Additional Derivative Topics:**

(Derivatives of Exponential and Logarithmic Functions, The Chain Rule, Implicit Differentiation, Elasticity of Demand)

1. Find the derivative of each function.

- |                                     |   |                                  |                                  |
|-------------------------------------|---|----------------------------------|----------------------------------|
| a) $y = \ln x^{10} + e^x - 3x^e$    | b) $y = xx^e + ee^x$                      | c) $y = 2^x - 3x^2$              | d) $f(x) = -\log_2 x + 10 \ln x$ |
| e) $h(t) = \frac{t \ln t}{e^t}$     | f) $y = \frac{\log_2 x}{1+x^2}$           | g) $y = \frac{10^x}{1+x^4}$      | i) $f(x) = 4x^3 \log_4 x^3$      |
| j) $f(u) = \frac{u^2 e^u}{1+\ln u}$ | k) $y = 2 \ln(x^2 - 3x + 4)$              | l) $y = e^{x^2+3x+1}$            | m) $y = (x^4 + 3)^{1/2}$         |
| n) $f(x) = (2^x + \log_2 x)^2$      | o) $f(x) = \ln(x^2 + 3)^{3/2}$            | p) $f(x) = [\ln(x^2 + 3)]^{3/2}$ | r) $f(x) = 3x^3(x^2 + 1)^3$      |
| s) $y = [\log_3(3x^2 - 1)]^4$       | t) $y = \frac{(3x^2 - 7)^5}{2\sqrt{x^3}}$ |                                  |                                  |

2. Find the equation(s) of the tangent line(s) to the graph of  $x^2 + y^2 - xy - 7 = 0$  at  $x = 1$ .

3. Find  $x'$  for  $x = x(t)$  defined implicitly by

$$1 + x \ln t = te^x$$

and evaluate  $x'$  at  $(t, x) = (1, 0)$ .

4. Use implicit differentiation to find  $y'$  and evaluate  $y'$  at the indicated point.

- |                                  |                                 |                                 |                                     |
|----------------------------------|---------------------------------|---------------------------------|-------------------------------------|
| a) $2xy + y + 2 = 0$ ; $(-1, 2)$ | b) $x^3 - y = \ln y$ ; $(1, 1)$ | c) $e^y = x^2 + y^2$ ; $(1, 0)$ | d) $x \ln y + 2y = 2x^3$ ; $(1, 1)$ |
|----------------------------------|---------------------------------|---------------------------------|-------------------------------------|

5. For the demand equation

$$x = \sqrt[3]{1500 - p^3}$$

find the rate of change of  $p$  with respect to  $x$  by differentiating implicitly ( $x$  is the number of items that can be sold at a price of  $\$p$ ).

6. The price  $p$  and the demand  $x$  for a product are related by the price–demand equation

$$x = f(p) = 1000(40 - p).$$

Find and interpret each of the following:

- |           |            |            |
|-----------|------------|------------|
| a) $E(8)$ | b) $E(30)$ | c) $E(20)$ |
|-----------|------------|------------|

7. A manufacturer of sunglasses currently sells one type for \$21 a pair. The price  $p$  and the demand  $x$  for these glasses are related by

$$x = f(p) = 9500 - 250p.$$

If the current price is increased, will revenue increase or decrease?

8. Given the price–demand equation

$$p + 0.005x = 30.$$

- a) Express the demand  $x$  as a function of the price  $p$ .
- b) Find the elasticity of demand,  $E(p)$ .
- c) What is the elasticity of demand when  $p = \$10$ ? If this price is increased by 10%, what is the approximate percentage change in demand?
- d) What is the elasticity of demand when  $p = \$25$ ? If this price is increased by 10%, what is the approximate percentage change in demand?
- e) What is the elasticity of demand when  $p = \$15$ ? If this price is increased by 10%, what is the approximate percentage change in demand?

9. Given the price–demand equation

$$p + 0.01x = 50.$$

- a) Express the demand  $x$  as a function of the price  $p$ .
- b) Find the elasticity of demand,  $E(p)$ .
- c) What is the elasticity of demand when  $p = \$10$ ? If this price is increased by 5%, what is the approximate percentage change in demand?
- d) What is the elasticity of demand when  $p = \$45$ ? If this price is increased by 5%, what is the approximate percentage change in demand?
- e) What is the elasticity of demand when  $p = \$25$ ? If this price is increased by 5%, what is the approximate percentage change in demand?

10. Given the price–demand equation

$$p + 0.02x = 60.$$

- a) Express the demand  $x$  as a function of the price  $p$ .
- b) Express the revenue  $R$  as a function of the price  $p$ .
- c) Find the elasticity of demand,  $E(p)$ .
- d) For which values of  $p$  is demand elastic? Inelastic?
- e) For which values of  $p$  is revenue increasing? Decreasing?
- f) If  $p = \$10$  and the price is decreased, will revenue increase or decrease?
- g) If  $p = \$40$  and the price is decreased, will revenue increase or decrease?

11. The price–demand equation for hamburgers at a fast-food restaurant is

$$x + 400p = 3000.$$

Currently, the price of a hamburger is \$3.00. If the price is increased by 10%, will revenue increase or decrease?