

**Faculty of Engineering**  
**Mathematical Analysis I**  
**Fall 2018**  
**Exercises 1: Functions**

1. Find the domain of the each function.<sup>1</sup>

a)  $f(x) = e^x + \sqrt{x+2} + \frac{1}{\ln(1-x)}$

b)  $f(x) = \frac{1}{x^2-4} + \frac{\sqrt[3]{x-2}}{x^2+1} + 2^{x+\sqrt{x-1}}$

c)  $f(x) = \frac{\log_2(x+1)}{\sqrt{3-|x-1|}} + \cos x$

d)  $f(x) = 5\sqrt{|x-2|-|6-x|} + \lfloor x^2 \rfloor + \lceil x \rceil^2$

2. Give an example of each type of function: Power function, root function, polynomial, rational function, algebraic function, trigonometric function, exponential function, logarithmic function and transcendental function.

3. Determine whether  $f$  is even, odd, or neither even nor odd.

(a)  $f(x) = x + \sqrt{1+x^2}$

(b)  $f(x) = \frac{e^x + 1}{e^x - 1}x$

(c)  $f(x) = \log(x + \sqrt{1+x^2})$

(d)  $f(x) = \log \frac{1+x}{1-x}$

4. If  $f(x) = \ln x$  and  $g(x) = x^2 - 9$ , find the functions  $f \circ g$ ,  $g \circ f$ ,  $f \circ f$ ,  $g \circ g$ , and their domains.

5. Show that  $f(x) = x^2 - 4x + 5$  is decreasing on  $(-\infty, 2]$  and increasing on  $[2, \infty)$ .

6. If

$$f(x) = \begin{cases} 4x - 3, & x \geq 0, \\ x^2 - 2x - 6, & x < 0 \end{cases}$$

and

$$g(x) = x^2 - 1.$$

Find  $(f + g - f \circ g \circ f)(-1)$ .

7. Use transformations to sketch the graph of the function roughly by hand. On what interval is  $f$  increasing or decreasing?

(a)  $f(x) = 2 - \sqrt{-x}$

(b)  $f(x) = (x+1)^{1/3} - 5$

(c)  $f(x) = |x^2 - 4| + 1$

(d)  $f(x) = -2(x+1)^3 - 3$

(e)  $f(x) = 1 - 3 \ln(x-2)$

(f)  $f(x) = 3 - 2^{-x+1}$

(g)  $f(x) = \begin{cases} 1+x, & x < 0, \\ e^x, & x \geq 0 \end{cases}$

(h)  $f(x) = 1 - \sin 2x$

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<sup>1</sup> $\lfloor \cdot \rfloor$  and  $\lceil \cdot \rceil$  stand for the greatest integer and the least integer functions, respectively.

(i)  $f(x) = \frac{1}{1+x} - 1$

8. The table shows the electricity rates charged by Easton Utilities in the summer months. Write a piecewise definition of the monthly charge  $S(x)$  (in dollars) for a customer who uses  $x$  kWh in a summer month and graph the function  $S(x)$  roughly.

**Table: Energy Charges**

|                                      |
|--------------------------------------|
| \$3.00 for the first 20 kWh or less  |
| \$5.70 per kWh for the next 180 kWh  |
| \$3.46 per kWh for the next 800 kWh  |
| \$2.17 per kWh for all over 1000 kWh |

9. Trussville Utilities uses the rates shown in the following table to compute the monthly cost of natural gas for residential customers. Write a piecewise definition for the cost of consuming  $x$  CCF of natural gas and graph the function.

**Table: Charges per Month**

|                                       |
|---------------------------------------|
| \$0.7675 per CCF for the first 50 CCF |
| \$0.6400 per CCF for the next 150 CCF |
| \$0.6130 per CCF for all over 200 CCF |

10. A personal-computer salesperson receives a base salary of \$1000 per month and a commission of 5% of all sales over \$10000 during the month. If the monthly sales are \$20000 or more, then the salesperson is given an additional \$500 bonus. Let  $E(s)$  represent the person's earnings per month as a function of the monthly sales  $s$ .

- (a) Write a piecewise definition of the function  $E(s)$  and evaluate  $E(25000)$ .  
(b) Graph  $E(s)$  for  $0 \leq s \leq 30000$ .

11. Determine whether the statement is true or false. If it is true, explain why. If it is false, give an example that disproves the statement.

- (a) If  $f$  and  $g$  are even, then  $f + g$  is even.  
(b) If  $f$  and  $g$  are odd, then  $f + g$  is odd.  
(c) If  $f$  and  $g$  are odd, then  $fg$  is even.  
(d) If  $g$  is even, then  $f \circ g$  is even.  
(e) If  $g$  is odd, then  $f \circ g$  is odd.  
(f) If  $f$  is a function, then  $f(s+t) = f(s) + f(t)$ .  
(g) A vertical line intersects the graph of a function at most once.  
(h) If  $f(s) = f(t)$ , then  $s = t$ .  
(i) If  $f$  and  $g$  are functions, then  $f \circ g = g \circ f$ .