Faculty of Engineering
Mathematical Analysis I
Fall 2018
Exercises 3: *Derivative*

1. Calculate y'.

   (a) $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$

   (b) $y = \dfrac{3^{x^2+1}}{1+x^2}$

   (c) $y = \dfrac{1}{\sin{(x - \sin x)}}$

   (d) $xy^4 + x^2 y = x + 3y$

   (e) $x^2 \cos y + \sin 2y = xy$

   (f) $y = \arctan{(\arcsin{\sqrt{x}})}$

   (g) $y = (\cos x)^x$

   (h) $y = x^{\sin x} + \ln\left(x^{\sqrt{x}}\right)$

   (i) $y = \dfrac{(x+3)(x+4)(x+5)}{(x+6)(x+7)}$

   (j) $y = \sin\left(\cos^2\left(\tan x^2\right)\right)$

   (k) $y = \log_4\left(\cos\frac{1}{x}\right)$

   (l) $y = (x-1)(x-2)(x-3)\cdots(x-101)$, at the point $x = 3$.

   (m) $y = x\,|x|$, at the point $x = 0$.

   (n) $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$, at the point $x = 0$.

2. Let $f : [2, \infty) \longrightarrow \mathbb{R}$ be a function defined by $f(x) = x^2 - 6x + 3$. Find $\left(f^{-1}\right)'(-2)$.

3. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by $f(x) = e^{2x} + 3x - 2$. Find
$$\lim_{x \to 0} \frac{f^2(x) - f^2(0)}{x}.$$

4. Find values of a and b that make
$$f(x) = \begin{cases} \sqrt{x}, & x \leq 1, \\ ax^2 + b, & x > 1 \end{cases}$$

   differentiable at $x = 1$.

5. Let
$$f(x) = \begin{cases} \cos\frac{\pi}{4}x, & x \leq 1, \\ ax + b, & x > 1 \end{cases}.$$

   (a) Determine the values of a and b so that f is continuous everywhere.

   (b) Determine the values of a and b so that f is differentiable everywhere.

6. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined as
$$f(x) = \begin{cases} e^x - 1, & x < 0, \\ \sin(x) + x^2, & x \geq 0 \end{cases}.$$

   Say where f is continuous and differentiable.

7. Let $f(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ 5 - x, & \text{if } 0 < x < 4 \\ \dfrac{1}{5-x}, & \text{if } x \geq 4 \end{cases}.$

   (a) Sketch the graph of f. Say the domain and range of f.

(b) Find $f'_- (4)$ and $f'_+ (4)$.

(c) Where is f discontinuous? Explain your answer.

(d) Where is f not differentiable? Explain your answer.

8. Let $f(x) = \begin{cases} (x-1)^2, & x \leq 0, \\ \sqrt{x}, & 0 < x < 4 \\ \frac{x}{2}, & 4 < x < 6 \\ 3, & x = 4 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Find the numbers at which f is not continuous. Explain your answer.

(c) Find the numbers at which f is not differentiable. Explain your answer.

9. Let $f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0, \\ 3 - x, & \text{if } 0 \leq x < 3 \\ (x-3)^2, & \text{if } x > 3 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Find the numbers at which f is not continuous. Explain your answer.

(c) Find the numbers at which f is not differentiable. Explain your answer.

10. Let $f(x) = \begin{cases} 1 + x^2, & \text{if } x \leq 0, \\ 2 - x, & \text{if } 0 < x \leq 2 \\ (x-2)^2, & \text{if } x > 2 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Find the numbers at which f is not continuous. Explain your answer.

(c) Find the numbers at which f is not differentiable. Explain your answer.

11. Let $f(x) = \begin{cases} x + 1, & \text{if } x \leq 1, \\ \dfrac{1}{x}, & \text{if } 0 < x < 3 \\ \sqrt{x - 3}, & \text{if } x > 3 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Find the numbers at which f is not continuous. Explain your answer.

(c) Find the numbers at which f is not differentiable. Explain your answer.

12. Let $f(x) = \begin{cases} \dfrac{x}{2}, & x \neq 1, \\ 1, & x = 1 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Does $\lim_{x \to 1} f(x)$ exists? Explain your answer.

(c) Is f continuous at $x = 1$? Explain your answer.

(d) Is f differentiable at $x = 1$? Explain your answer.

13. Let $f(x) = \begin{cases} |x - 1|, & x \leq 1, \\ \sqrt{x - 1}, & x > 1 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Is f continuous at $x = 1$? Explain your answer.

(c) Is f differentiable at $x = 1$? Explain your answer.

14. Let $f(x) = \begin{cases} x^{1/3}, & x \leq 1, \\ -|x - 1|, & x > 1 \end{cases}$.

(a) Sketch the graph of f. Find the domain and range of f.

(b) Is f continuous at $x = 0$ or $x = 1$? Explain your answer.

(c) Is f differentiable at $x = 0$ or $x = 1$? Explain your answer.

15. Find an equation of the tangent line to the curve $x \cos x + \sin y = \frac{1}{2}$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$.

16. Find an equation of the tangent line to the curve $e^{xy} + y^2 \sin(\pi x) = e$ at the point $(1, 1)$.

17. Find an equation of the straight line that passes through the point $(-2, 0)$ and is tangent to the curve $y = \sqrt{x}$.