Faculty of Engineering
Mathematical Analysis I
Fall 2018
Exercises 5
Mean Value Theorem, Techniques of Integration

1. Using Mean Value Theorem show that $\sqrt{1+x} < 1 + \frac{x}{2}$ for $x > 0$ and for $-1 \leq x < 0$.
2. Show that $\tan x > x$ for $0 < x < \pi/2$.
3. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be? (Hint: Use MVT)
4. Evaluate the following indefinite integrals
   (a) $\int \frac{\sin(3\ln x)}{x} dx$
   (b) $\int e^x \sqrt{1+e^x} dx$
   (c) $\int x^3 \cos\left(x^2\right) dx$
   (d) $\int \frac{e^{\arctan x}}{1+x^2} dx$
   (e) $\int \cos^6 x dx$
   (f) $\int \sin^3 x \cos^2 x dx$
   (g) $\int \sin^6 x \cos^3 x dx$
   (h) $\int \sin^2 x \cos^2 x dx$
   (i) $\int \tan^3 x \sec^7 x dx$
   (j) $\int \tan^2 x dx$
   (k) $\int \tan^5 x dx$
   (l) $\int \cot^3 x \csc^3 x dx$
   (m) $\int \cot^6 x \csc^4 x dx$
   (n) $\int \cos x \cos^5(\sin x) dx$
   (o) $\int \frac{x^2}{\sqrt{9-x^2}} dx$
   (p) $\int \frac{1}{(4+x^2)^{3/2}} dx$
   (q) $\int \frac{\sqrt{x^2-1}}{x} dx$
   (r) $\int \frac{\sqrt{x-1}+1}{\sqrt[3]{x-1}} dx$
   (s) $\int \frac{x^3}{(4x^2+9)^{3/2}} dx$
   (t) $\int \frac{x}{\sqrt{3-2x-x^2}} dx$
   (u) $\int e^x \cos x dx$
   (v) $\int x^2 e^x dx$
   (w) $\int x^2 \ln x dx$
   (x) $\int x \arctan x dx$
   (y) $\int \frac{x^4+2x^2+x}{x^3+1} dx$
   (z) $\int \frac{3x-1}{x^2-2x-3} dx$
   (ξ) $\int \frac{x+3}{x^4+9x^2} dx$
5. Write out the form of the partial fraction decomposition of the functions
   (a) $\frac{2x+1}{(x+1)^3(x^2+4)^2}$ (b) $\frac{x^4}{(x^3+x)^3(x^2-x+3)}$