(b) $\lim_{x\to 0^+} x^x = (0^0)$

$$y = x^x \Rightarrow \ln y = x\ln x \Rightarrow y = e^{x\ln x}$$

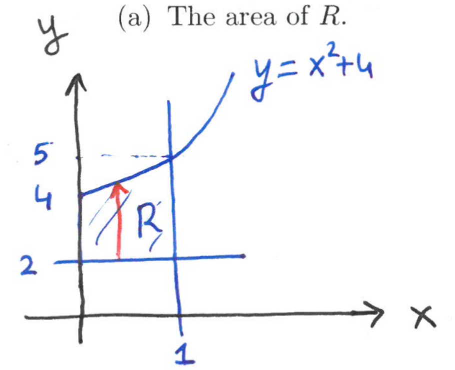
$$\lim_{x\to 0^+} x\ln x = [0\cdot\infty] = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} \underset{LH}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+}(-x) = 0$$

$$\therefore \lim_{x\to 0^+} x^x = e^0 = 1$$

(30 point) 6. Let R be the region bounded by the curves $y = x^2+4$, $y=2$, $x=0$ and $x=1$. Set up, **but do not evaluate**, an integral for;

(a) The area of R.



$$A(R) = \int_0^1 [(x^2+4)-2]\,dx$$

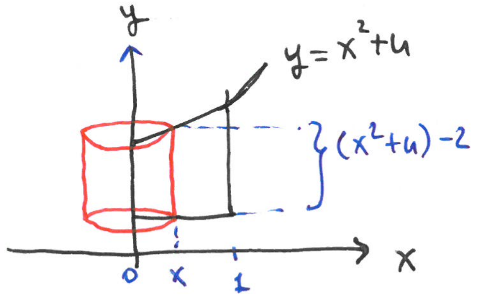
(b) The perimeter of R.

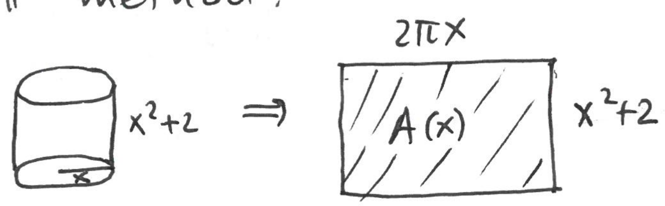
$$P(R) = 2+1+3+\int_0^1 \sqrt{1+[(x^2+4)']^2}\,dx$$

$$= 6+\int_0^1\sqrt{1+4x^2}\,dx$$

(c) The volume obtained by rotating R about the y-axis.

Apply cylindrical shell method:





$$V = \int_0^1 A(x)\,dx = 2\pi\int_0^1 x(x^2+2)\,dx$$

İstanbul Commerce University
Faculty of Engineering
MAT121-Mathematical Analysis I
Resit Exam

Name-Surname: ANSWER KEY
Student Number:
Department:

Abdullah YENER
25.01.2018, 11:00
C-307

Attention. The test duration is 90 minutes. The use of a calculator, cell phone or other equivalent electronic devices or documents are not allowed. Show your work in a reasonable detail. A correct answer without proper or too much reasoning may not get any credit. Good luck.

(15 point) 1. A spherical balloon is inflated with helium at the rate of 100π ft^3/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft?

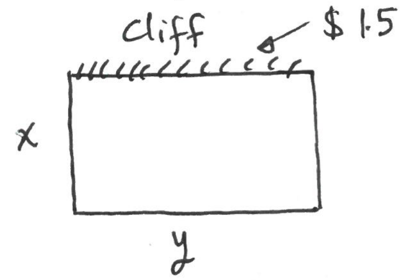
Given: $\frac{dV}{dt} = 100\pi$ ft^3/min, Unknown: $\frac{dr}{dt} = ?$ when $r = 5$ ft

Equation: $V = \frac{4}{3}\pi r^3$

Differentiate wrt t:

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}\ ; \quad 100\pi = 4\pi 5^2\frac{dr}{dt} \Rightarrow \boxed{\frac{dr}{dt} = 1 \text{ ft}^3/\text{min}}$$

(15 point) 2. A rancher wishes to build a rectangular corral of 128000 ft^2 with one side along a vertical cliff. The fencing along the cliff cost \$ 1.5 per foot, whereas along the other three sides the fencing cost \$ 2.5 per foot. Find the dimensions of the corral so that the cost of fencing is minimum.



$$\text{Area} = xy = 128000 \Rightarrow y = \frac{128000}{x}$$

$$\text{Cost}(C) = (2x+y)\,2.5 + 1.5y = 5x+4y$$

$$C(x) = 5x + 4\frac{128000}{x} = 5x + \frac{512000}{x}$$

$$C'(x) = 5 - \frac{512000}{x^2}$$

$$C'(x) = 0 \Rightarrow x^2 = \frac{512000}{5} = 102400$$

$$x = \sqrt{102400} = 320 \text{ m}$$

$$y = \frac{128000}{320} = 400 \text{ m}$$

(30 point) 3. Let $f(x) = \frac{e^x}{x}$.

(a) Find the domain and intercepts of f.

$Dom f = \mathbb{R} \setminus \{0\}$, No intercepts.

(b) Find all asymptotes of the graph of f.

- $x=0$ is a V.A. : $\lim_{x\to 0^-} \frac{e^x}{x} = -\infty$, $\lim_{x\to 0^+} \frac{e^x}{x} = \infty$
- $y=0$ is a H.A. ; $\lim_{x\to -\infty} \frac{e^x}{x} = 0$, $\lim_{x\to \infty} \frac{e^x}{x} = \infty$

(c) Find intervals of increase and decrease, and the local extrema of f.

$$f'(x) = \frac{e^x x - e^x}{x^2} = \frac{e^x(x-1)}{x^2} = 0 \iff x = 1 \text{ critical point}$$

x		0		1	
f'	$-$		$-$	0	$+$
f	$\searrow$		$\searrow$		$\nearrow$

increasing on $[1,\infty)$
decreasing on $(-\infty,0)\cup(0,1]$

$f(1)=e$ is a local min., no loc. max.

(d) Find the intervals of concavity and the inflection points of f.

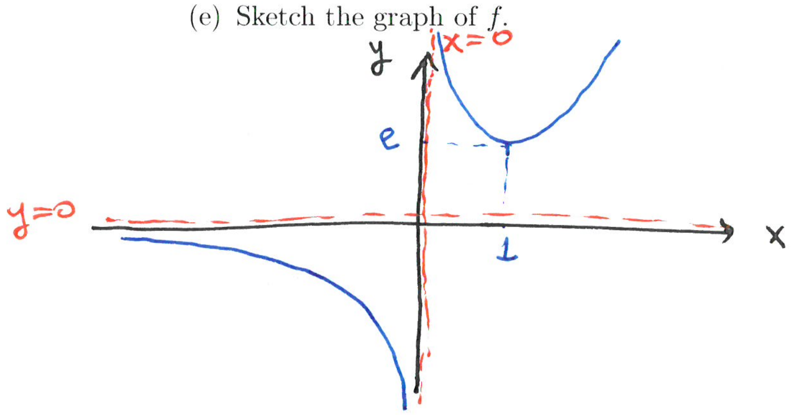
$$f''(x) = \frac{[e^x(x-1)+e^x]x^2 - e^x(x-1)2x}{x^4} = \frac{e^x(x^2-2x+2)}{x^3}$$

$f''(x)=0$; no soln.

x		0	
f''	$-$		$+$
f	$\cap$ CD		$\cup$ CU

The graph is CD on $(-\infty,0)$
The graph is CU on $(0,\infty)$
No inflenction point, since $0 \notin Dom f$

(e) Sketch the graph of f.



(30 point) 4. Evaluate the following integrals.

(a) $\int x \sin x\,dx$ (Apply integration by parts)

$u = x \Rightarrow du = dx$
$dv = \sin x\,dx \Rightarrow v = \int \sin x\,dx = -\cos x$

$$\therefore \int x\sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$$

(b) $\int_1^e \frac{\cos(3\ln x)}{x}dx$ (Apply substitution)

$u = \ln x \Rightarrow du = \frac{dx}{x}$
$x=1 \Rightarrow u=0$
$x=e \Rightarrow u=1$

$$\therefore \int_1^e \frac{\cos(3\ln x)}{x}dx = \int_0^1 \cos 3u\,du = \frac{1}{3}\sin 3u\Big|_0^1$$
$$= \frac{1}{3}\sin 3$$

(c) $\int \frac{3x-1}{x^2-2x-3}dx$ (Partial Fraction)

$$\frac{3x-1}{(x-3)(x+1)} = \frac{A}{x-3}_{(x+1)} + \frac{B}{x+1}_{(x-3)}$$; $A(x+1) + B(x-3) = 3x-1$

Put $x=3$, then $4A = 8 \Rightarrow A = 2$.
Put $x=-1$, then $-4B = -4 \Rightarrow B = 1$

$$\int \frac{3x-1}{x^2-2x-3}dx = 2\int\frac{dx}{x-3} + \int\frac{dx}{x+1}$$
$$= 2\ln|x-3| + \ln|x+1| + C$$

(20 point) 5. Evaluate the following limits (You can use the L'Hospital Rule).

(a) $\lim_{x\to 0} \frac{\int_0^x \sin(t^3)\,dt}{x} = \left[\frac{0}{0}\right] \underset{LH}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\left(\int_0^x \sin t^3\,dt\right)}{1}$

$\underset{FTC}{=} \lim_{x\to 0} \frac{\sin x^3}{1} = \frac{0}{1} = 0$