İstanbul Commerce University
Numerical Analysis
Summer School
Sample Final Exam

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Attention. The test duration is 110 minutes. The use of a calculator is allowed but cell phone or other equivalent electronic devices or documents are not allowed. Show your work in a reasonable detail. A correct answer without proper or too much reasoning may not get any credit. Good luck.

1. (a) Find the $P_2(x)$ Lagrange polynomial interpolating the function $f(x) = \sin \frac{\pi x}{2}$ at $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$.
   (b) Give an error bound for $|f(x) - P_2(x)|$.

2. (a) According to following datas

$$\begin{array}{rcccc} x_i: & -1 & 0 & 2 & 3 \\ f(x_i): & -1 & 3 & 11 & 27 \end{array}$$

   find the $P_3(x)$ Lagrange interpolation polynomial.
   (b) Find $P_3(-2)$.

3. (a) Fill the following Divided Difference Table

x_i	$f(x_i)$	1^{st}	2^{nd}	3^{rd}
$x_0 = -2$	-39			
$x_1 = -1$	1			
$x_2 = 0$	1			
$x_3 = 1$	3			

   (b) Find the $P_3(x)$ Newton interpolating polynomial using the part a). What is the value of $P_3\left(\frac{1}{2}\right)$?
   (c) Find the quadratic Newton polynomial $P_2(x)$ interpolating $f(x)$ at x_1, x_2 and x_3.

4. Using the method of least squares, fit a straight line to the four points given in the following table

$$\begin{array}{rcccc} x_i: & 0 & 2 & 3 & 5 \\ f(x_i): & 2 & 0 & -2 & -3 \end{array}$$

5. Use one step of Newton-Raphson method to solve the systems of nonlinear equations

$$f_1(x, y) = 5x - 15y^2 = 0,$$
$$f_2(x, y) = \ln \sqrt{x} - \ln y - \frac{1}{2} = 0.$$

   Take the initial point as $\left(x^{(0)}, y^{(0)}\right)^T = (5, 1)^T$.

6. (a) Use the composite trapezoidal rule with $n = 4$ to approximate the integral

$$\int_0^{\pi} e^x \cos x dx.$$

   (b) Give an upper bound for the error involved in this approximation.

7. (a) Use the composite Simphson's rule with $n = 4$ to approximate the integral

$$\int_0^1 e^{x^2} dx.$$

   (b) How large should n be to guarantee that the composite Simphson's rule approximation to the integral in part a) is accurate to within 0.0001?