İstanbul Commerce University
Numerical Analysis
Summer School
Sample Midterm Exam

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Attention. The test duration is 100 minutes. The use of a calculator is allowed but cell phone or other equivalent electronic devices or documents are not allowed. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. Good luck.

(10) 1. Let $X_A = 11.33$ and $Y_A = 2.15$, and let absolute error bounds for X and Y be $|e_X| \leq 0.005$ and $|e_Y| \leq 0.005$, respectively. Give a relative error bounds for $x + xy$.

(10+10) 2. (a) Find the 2^{nd} Taylor polynomial of the function $f(x) = \sqrt{x}$ about $x = 25$.

(b) Use part (a) to approximate $\sqrt{26}$.

(5+5+5+5) 3. Let A be a 3×3 matrix given by

$$A = \begin{pmatrix} 60 & 30 & 20 \\ 30 & 25 & 15 \\ 20 & 15 & 12 \end{pmatrix}.$$

Give the LU decomposition of A;

(a) By using the Doolittle's factorization.

(b) By using the Crout's factorization.

(c) By using the Cholesky factorization.

(d) Use part (a), (b) or (c) to solve the following system of equations

$$\begin{aligned} 60x_1 + 30x_2 + 20x_3 &= 1 \\ 30x_1 + 25x_2 + 15x_3 &= \frac{5}{2} \\ 20x_1 + 15x_2 + 12x_3 &= 3 \end{aligned}$$

(10+10) 4. (a) Use Gauss-Jordan method to find the inverse of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}.$$

(b) Use part (a) to solve the following system of equation $Ax = b$ where $b = (1, 2, 1)^T$.

(5+5+5+5) 5. Let the system of equation

$$\begin{aligned} 4x_1 + 2x_2 + x_3 &= 11 \\ -x_1 + 2x_2 &= 3 \\ 2x_1 + x_2 + 4x_3 &= 16 \end{aligned} \tag{1}$$

be given.

(a) Write the iteration matrix of the system (1) by using Richardson method.

(b) Write the iteration matrix of the system (1) by using Jacobi method.

(c) Write the iteration matrix of the system (1) by using Gauss-Seidel method.

(d) Apply one of the above methods for two iterations with the initial point $x = (1, 1, 1)^T$.

(5+5+5) 6. Consider the equation $x^2 - 3x + 2 = 0$. Starting with $x_1 = 1$ and $x_2 = 2$, compute x_2 and x_3;

(a) Using the Bisection method.

(b) Using the Newton-Rapson method.

(c) Using the Secant method.