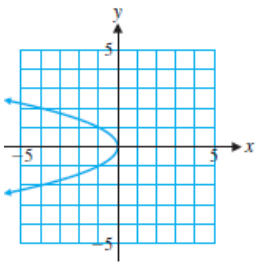
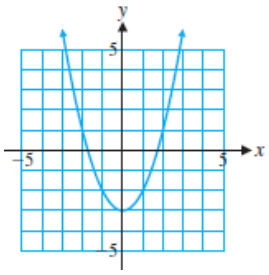
Faculty of Business Administration
MAT101-Mathematics I / 2019 Fall
Exercise-2: Elementary Functions, Transformations, Quadratic Functions, Polynomial and Rational Functions

1. Indicate whether each graph specifies a function:





2. Determine which of the following equations specify functions with independent variable x.

a) $y^2 - x^4 = 9$, x a real number **b)** $3y - 2x = 3$, x a real number

3. Find the domain of each function:

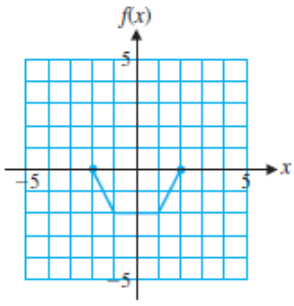
a) $h(x) = 7 - x^2 - 3x^5$ **b)** $k(x) = \sqrt{8 - x}$ **c)** $f(x) = \dfrac{2x - 5}{x^2 - x - 6}$ **d)** $g(x) = \dfrac{3x}{\sqrt{5 - x}}$

e) $l(x) = 20 - 10\sqrt[3]{x - 2}$

4. For $f(x) = 2x - 1$ and $g(x) = 2x^2 + 3x$, find:

a) $f(-12) + g(-22)$ **b)** $f(-2)g(0)$ **c)** $\dfrac{g(-1)}{f(2)}$ **d)** $\dfrac{f(2)}{g(-1)}$

5. Sketch a graph of each of the function in parts (a)–(d) using the graph of function f in the figure shown on right.



a) $y = -f(x)$ **b)** $y = f(x) + 4$ **c)** $y = f(x - 2)$ **d)** $y = -f(x + 3) - 3$

6. Indicate verbally how the graph of each function is related to the graph of one of the six basic functions. Using transformation sketch a graph of each function.

a) $h(x) = -|x + 3|$ **b)** $f(x) = -|x - 5|$ **c)** $g(x) = (x - 2)^2 + 5$ **d)** $k(x) = 5 - \sqrt{x}$ **e)** $l(x) = -3 + \sqrt[3]{x}$

7. The graph of the function g is formed by applying the indicated sequence of transformations to the given function f. Find an equation for the function g and graph g.

a) The graph of $f(x) = \sqrt{x}$ is shifted 2 units to the right and 3 units down.

b) The graph of $f(x) = |x|$ is shifted 3 units to the left and 2 units up.

c) The graph of $f(x) = x^2$ is reflected in the x axis and shifted to the left 2 units and up 4 units.

d) The graph of $f(x) = x^3$ is reflected in the x axis and shifted 2 units to the right and down 1 unit.

8. Graph each function.

a) $f(x)=\begin{cases}2-2x & \text{if } x<2\\ x-2 & \text{if } x\geq 2\end{cases}$

b) $g(x)=\begin{cases}4x+20 & \text{if } 0\leq x\leq 20\\ 2x+60 & \text{if } 20<x\leq 100\\ -x+360 & \text{if } x>100\end{cases}$

8. Using statistical methods, the financial department of a hospital arrived at the cost equation

$$C(x)=0.00048(x-500)^3+60000 \qquad 100\leq x\leq 1000$$

where C(x) is the cost in dollars for handling x cases per month.

a) Describe how the graph of function C can be obtained from the graph of one of the basic functions.

b) Sketch a graph of function C using part a).

9. Trussville Utilities uses the rates shown in Table 2 to compute the monthly cost of natural gas for residential customers. Write a piecewise definition for the cost of consuming x CCF of natural gas and graph the function.

Table 2 **Charges per Month**

\$0.7675 per CCF for the first 50 CCF
\$0.6400 per CCF for the next 150 CCF
\$0.6130 per CCF for all over 200 CCF

10. Table 3 shows the electricity rates charged by Monroe Utilities in the summer months. The base is a fixed monthly charge, independent of the kWh (kilowatt-hours) used during the month for a customer who uses x kWh in a summer month.

Table 3 **Summer (July–October)**

Base charge, \$8.50
First 700 kWh or less at 0.0650/kWh
Over 700 kWh at 0.0900/kWh

a) Write a piecewise definition of the monthly charge S(x). **b)** Graph S(x).

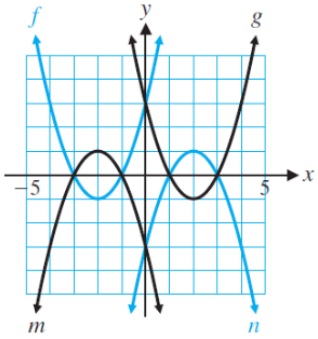
11. A personal-computer salesperson receives a base salary of \$1000 per month and a commission of 5% of all sales over \$10000 during the month. If the monthly sales are \$20000 or more, then the salesperson is given an additional \$500 bonus. Let E(x) represents the person's earnings per month as a function of the monthly sales x.

a) Write a piecewise definition of the function E(x). **b)** Graph E(x).

12. Find the vertex form of each quadratic function by completing the square then sketch the graph of it.

a) $f(x)=x^2+16x$ **b)** $f(x)=-2x^2+4x-5$ **c)** $f(x)=x^2-12x-8$ **d)** $f(x)=3x^2+18x+21$

13. Match each equation with a graph of one of the functions f, g, m or n in the figure.



1. $y=-(x+2)^2+1$ _____

2. $y=(x-2)^2-1$ _____

3. $y=(x+2)^2-1$ _____

4. $y=-(x-2)^2+1$ _____

14. Given the quadratic function $f(x) = -4x^2 + 16x - 15$.

a) Find the vertex form for f.

b) Find the intercepts, vertex and the maximum or minimum. State the range of f.

c) Graph function f.

15. Given the quadratic function $g(x) = 0.5x^2 + 4x + 10$.

a) Find the vertex form for g.

b) Find the intercepts, vertex and the maximum or minimum. State the range of g.

c) Graph function g.

16. The marketing research department for a company that manufactures and sells notebook computers established the following price–demand and revenue functions:

$$p(x) = 2000 - 60x$$

$$R(x) = xp(x) = x(2000 - 60x)$$

where p(x) is the wholesale price in dollars at which x thousand computers can be sold, and R(x) is in thousands of dollars. Both functions have domain $1 \le x \le 25$.

a) Sketch a graph of the revenue function in a rectangular coordinate system.

b) Find the value of x that will produce the maximum revenue. What is the maximum revenue to the nearest thousand dollars?

c) What is the wholesale price per computer (to the nearest dollar) that produces the maximum revenue?

17. Use the revenue function from Problem 15, and the given cost function:

$$R(x) = x(2000 - 60x)$$

$$C(x) = 4000 + 500x$$

where x is thousands of computers, and C(x) and R(x) are in thousands of dollars. Both functions have domain $1 \le x \le 25$.

a) Sketch a graph of both functions in the same rectangular coordinate system. Interpret the graphs.

b) Find the break-even points.

c) For what values of x will a loss occur? A profit?

d) Sketch a graph of the profit function in a rectangular coordinate system. Interpret the graph.

18. For each polynomial function below find the degree, all x and y intercepts.

a) $f(x) = x^2 + 3x + 2$ **b)** $f(x) = x^4(x - 1)$ **c)** $f(x) = x^2 - 4x - 5$ **d)** $f(x) = \left(x^2 - 4\right)(x^3 + 27)$

19. For each rational function below find the intercepts for the graph, determine the domain and find any vertical or horizontal asymptotes for the graph.

a) $f(x) = \dfrac{x+2}{x-2}$ **b)** $g(x) = \dfrac{x^2 - x - 6}{x^2 - 3x - 10}$ **c)** $h(x) = \dfrac{x^2 + 3x}{x^3 - 36x}$ **d)** $k(x) = \dfrac{x^2 + 6x + 5}{x - 5}$

20. A company manufacturing snowboards has fixed costs of $200 per day and total costs of $3,800 per day at a daily output of 20 boards.

a) Assuming that the total cost per day, C(x), is linearly related to the total output per day, x, write an equation for the cost function.

b) The average cost per board for an output of x boards is given by $\bar{C}(x) = \frac{C(x)}{x}$. Find the average cost function.

c) Sketch a graph of the average cost function, including any asymptotes, for $1 \le x \le 30$.

d) What does the average cost per board tend to as production increases?