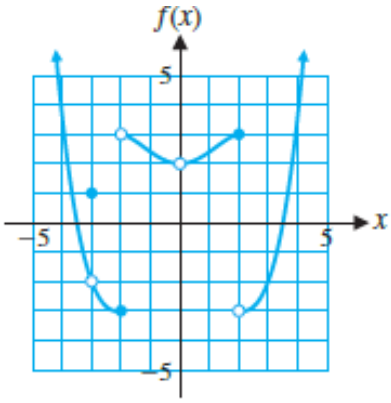
Faculty of Business Administration

MAT101-Mathematics I / 2019 Fall

Exercise-4: Limit and Continuity

1. Using the gragh of the following function $f(x)$ evaluate the indicated limits.



a) $lim_{x\to -3^+} f(x)$ b) $lim_{x\to -3^-} f(x)$ c) $lim_{x\to -3} f(x)$

d) $lim_{x\to -2^+} f(x)$ e) $lim_{x\to -2^-} f(x)$ f) $lim_{x\to -2} f(x)$

g) $lim_{x\to 0^+} f(x)$ h) $lim_{x\to 0^-} f(x)$ i) $lim_{x\to 0} f(x)$

j) $lim_{x\to 2^+} f(x)$ k) $lim_{x\to 2^-} f(x)$ l) $lim_{x\to 2} f(x)$

2. Find the limit or explain why the limit does not exist.

a) $lim_{x\to -3} \dfrac{x}{x+5}$ b) $lim_{x\to 0} \sqrt{16-7x}$ c) $lim_{x\to 1} \dfrac{|x-1|}{x-1}$ d) $lim_{x\to \sqrt{5}} \dfrac{x-\sqrt{5}}{|x-\sqrt{5}|}$

e) $lim_{x\to -3} \dfrac{x+3}{x^2+3x}$ f) $lim_{x\to -1} \dfrac{x^2-1}{(x+1)^2}$ g) $lim_{x\to 5} \dfrac{x-5}{x+2}$ h) $lim_{x\to 2} \dfrac{2x^2-3x-2}{x^2+x-6}$

3. Let $f(x)=\begin{cases} 1-x^2 & \text{if } x\le 0, \\ 1+x^2 & \text{if } x>0. \end{cases}$ Find each indicated quantity if it exists.

a) $lim_{x\to 0^+} f(x)$ b) $lim_{x\to 0^-} f(x)$ c) $lim_{x\to 0} f(x)$ d) $f(0)$

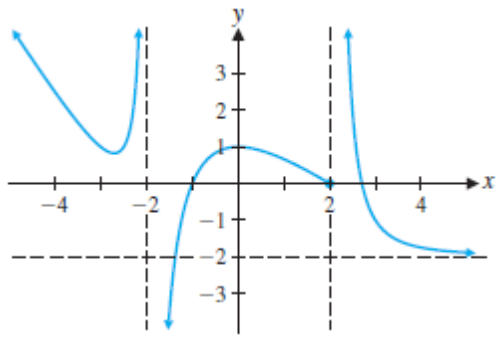
4. Let $f(x)=\begin{cases} x+3 & \text{if } x<-2, \\ \sqrt{x+2} & \text{if } x>-2. \end{cases}$ Find each indicated quantity if it exists.

a) $lim_{x\to -2^+} f(x)$ b) $lim_{x\to -2^-} f(x)$ c) $lim_{x\to -2} f(x)$ d) $f(-2)$

5. Let $f(x)=\begin{cases} \dfrac{x^2-9}{x+3} & \text{if } x<0, \\ \dfrac{x^2-9}{x-3} & \text{if } x>0. \end{cases}$ Find each indicated quantity if it exists.

a) $lim_{x\to -3} f(x)$ b) $lim_{x\to 3} f(x)$ c) $lim_{x\to 0^+} f(x)$ d) $lim_{x\to 0^-} f(x)$

6. Using the gragh of the following function $f(x)$ evaluate the indicated limits.



a) $\lim_{x\to\infty} f(x)$ b) $\lim_{x\to-\infty} f(x)$ c) $\lim_{x\to-2^+} f(x)$

d) $\lim_{x\to-2^-} f(x)$ e) $\lim_{x\to-2} f(x)$ f) $\lim_{x\to2^+} f(x)$

g) $\lim_{x\to2^-} f(x)$ h) $\lim_{x\to2} f(x)$

7. Find each limit. Use $-\infty$ and ∞ when appropriate.

a) $\lim_{x\to5^-} \dfrac{x}{x-5}$ b) $\lim_{x\to5^+} \dfrac{x}{x-5}$ c) $\lim_{x\to5} \dfrac{x}{x-5}$ d) $\lim_{x\to-2^-} \dfrac{x^2-3x+2}{x+2}$

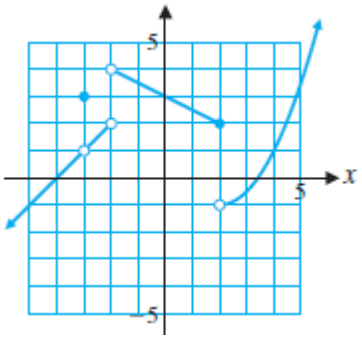
e) $\lim_{x\to-2^+} \dfrac{x^2-3x+2}{x+2}$ f) $\lim_{x\to-2} \dfrac{x^2-3x+2}{x+2}$ g) $\lim_{x\to\infty} \dfrac{2-3x^3}{7+4x^3}$

h) $\lim_{x\to-\infty} \dfrac{2-3x^3}{7+2x^3}$ i) $\lim_{x\to\infty} \dfrac{4x+11}{2x^3-5}$ j) $\lim_{x\to\infty} \dfrac{6x^5+2x}{9x^3+x^2}$ k) $\lim_{x\to-\infty} \dfrac{x^2-x+1}{4x^2}$

8. Find all horizontal and vertical asymptotes for the given functions.

a) $f(x)=\dfrac{2x}{x+2}$ b) $f(x)=\dfrac{x^2+1}{x^2-1}$ c) $f(x)=\dfrac{x^2-1}{x^2+2}$ d) $f(x)=\dfrac{2x^2-5x+2}{x^2-x-2}$

9. Use the graph to estimate the indicated function values and limits.



a) $\lim_{x\to-2^+} f(x)$ b) $\lim_{x\to-2^-} f(x)$ c) $\lim_{x\to-2} f(x)$

d) $f(-2)$ e) Is f continuous at $x=-2$? Why?

f) $\lim_{x\to-3^+} f(x)$ g) $\lim_{x\to-3^-} f(x)$ i) $\lim_{x\to-3} f(x)$

j) $f(-3)$ k) Is f continuous at $x=-3$? Why?

l) $\lim_{x\to2^+} f(x)$ m) $\lim_{x\to2^-} f(x)$ n) $\lim_{x\to2} f(x)$

o) $f(2)$ p) Is f continuous at $x=2$? Why?

10. Using the definition of continuity, discuss the continuity of each function at the indicated point.

a) $f(x)=x+1$ at $x=1$ b) $g(x)=\dfrac{x^2-1}{x-1}$ at $x=1$ c) $h(x)=\dfrac{x-2}{|x-2|}$ at $x=2$

d) $k(x)=\begin{cases} x+2 & \text{if } x\le 0, \\ 2-x & \text{if } x>0 \end{cases}$ at $x=0$

11. A company manufacturing snowboards has fixed costs of $200 per day and total costs of $3800 per day for a daily output of 20 boards.

a) Assuming that the total cost per day C(x) is linearly related to the total output per day x, write an equation for the cost function.

b) The average cost per board for an output of x boards is given by $\overline{C} = \frac{C(x)}{x}$. Find the average cost function.

c) What does the average cost per board tend to as production increases?

12. Natural-gas rates. Table 1 shows the rates for natural gas charged by the Middle Tennessee Natural Gas Utility District during summer months. The base charge is a fixed monthly charge, independent of the amount of gas used per month.

Table 1 **Summer (May–September)**

Base charge	$5.00
First 50 therms	0.63 per therm
Over 50 therms	0.45 per therm

a) Write a piecewise definition of the monthly charge S(x) for a customer who uses x therms in a summer month.

b) Graph S(x).

c) Is S(x) continuous at x = 50? Explain.

13. A personal-computer salesperson receives a base salary of $1000 per month and a commission of 5% of all sales over $10000 during the month. If the monthly sales are $20000 or more, then the salesperson is given an additional $500 bonus. Let E(s) represent the person's earnings per month as a function of the monthly sales s.

a) Write a piecewise definition of E(s).

b) Graph E(s) for $0 \le s \le 30000$.

c) Find $\lim_{s \to 10000} E(s)$ and E(10000). Is E(s) continuous at s = 10000? Explain.

d) Find $\lim_{s \to 20000} E(s)$ and E(20000). Is E(s) continuous at s = 20000? Explain.