MAT101 - Mathematics

Linear Equations and Inequalities

Linear Equations and Inequalities

The equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

and the inequality

$$\frac{x}{2} + 2(3x - 1) \geq 5$$

are both first degree in one variable. In general, a **first-degree**, or **linear**, **equation** in one variable is any equation that can be written in the form

$$\textbf{Standard form:} \quad ax + b = 0 \qquad a \neq 0 \tag{1}$$

If the equality symbol, $=$, in (1) is replaced by $<$, $>$, $\leq$, or $\geq$, the resulting expression is called a **first-degree**, or **linear**, **inequality**.

A **solution** of an equation (or inequality) involving a single variable is a number that when substituted for the variable makes the equation (or inequality) true. The set of all solutions is called the **solution set**. When we say that we **solve an equation** (or inequality), we mean that we find its solution set.

Linear Equations

Linear equations are generally solved using the following equality properties.

THEOREM Equality Properties

An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.
2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

Linear Equations

EXAMPLE Solving a Linear Equation Solve and check:

$$8x - 3(x - 4) = 3(x - 4) + 6$$

SOLUTION

$$8x - 3(x - 4) = 3(x - 4) + 6$$ Use the distributive property.

$$8x - 3x + 12 = 3x - 12 + 6$$ Combine like terms.

$$5x + 12 = 3x - 6$$ Subtract $3x$ from both sides.

$$2x + 12 = -6$$ Subtract 12 from both sides.

$$2x = -18$$ Divide both sides by 2.

$$x = -9$$

CHECK

$$8x - 3(x - 4) = 3(x - 4) + 6$$

$$8(-9) - 3[(-9) - 4] \stackrel{?}{=} 3[(-9) - 4] + 6$$

$$-72 - 3(-13) \stackrel{?}{=} 3(-13) + 6$$

$$-33 \stackrel{\checkmark}{=} -33$$

Matched Problem Solve and check: $3x - 2(2x - 5) = 2(x + 3) - 8$

Linear Equations

EXAMPLE 2 Solving a Linear Equation Solve and check: $\frac{x+2}{2} - \frac{x}{3} = 5$

SOLUTION we multiply both sides of the equation by 6:

$$6\left(\frac{x+2}{2} - \frac{x}{3}\right) = 6 \cdot 5$$

$$\overset{3}{\cancel{6}} \cdot \frac{(x+2)}{\underset{1}{\cancel{2}}} - \overset{2}{\cancel{6}} \cdot \frac{x}{\underset{1}{\cancel{3}}} = 30$$

$$3(x+2) - 2x = 30 \quad \text{Use the distributive property.}$$

$$3x + 6 - 2x = 30 \quad \text{Combine like terms.}$$

$$x + 6 = 30 \quad \text{Subtract 6 from both sides.}$$

$$x = 24$$

CHECK

$$\frac{x+2}{2} - \frac{x}{3} = 5$$

$$\frac{24+2}{2} - \frac{24}{3} \stackrel{?}{=} 5$$

$$13 - 8 \stackrel{?}{=} 5$$

$$5 \stackrel{\checkmark}{=} 5$$

Matched Problem 2 Solve and check: $\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$

Linear Equations

EXAMPLE Solving a Formula for a Particular Variable If you deposit a principal P in an account that earns simple interest at an annual rate r, then the amount A in the account after t years is given by $A = P + Prt$. Solve for

(A) r in terms of A, P, and t

(B) P in terms of A, r, and t

SOLUTION (A)

$$A = P + Prt$$ Reverse equation.

$$P + Prt = A$$ Subtract P from both sides.

$$Prt = A - P$$ Divide both members by Pt.

$$r = \frac{A - P}{Pt}$$

(B)

$$A = P + Prt$$ Reverse equation.

$$P + Prt = A$$ Factor out P (note the use of the distributive property).

$$P(1 + rt) = A$$ Divide by $(1 + rt)$.

$$P = \frac{A}{1 + rt}$$

Linear Inequalities

If a and b are real numbers, we write

$$a < b \quad a \text{ is less than } b$$

If $a < b$, we may also write

$$b > a \quad b \text{ is greater than } a.$$

THEOREM 2 Inequality Properties

An equivalent inequality will result, and the **sense or direction will remain the same** if each side of the original inequality

1. has the same real number added to or subtracted from it.
2. is multiplied or divided by the same *positive* number.

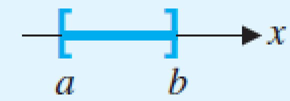
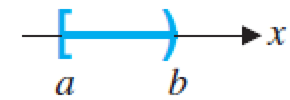
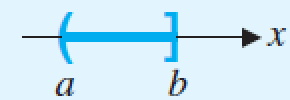
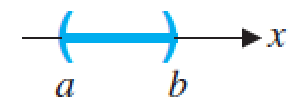
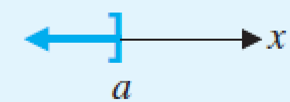
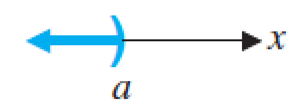
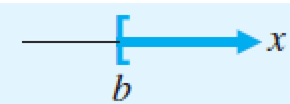
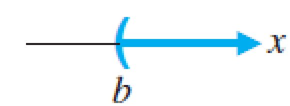
An equivalent inequality will result, and the **sense or direction will reverse** if each side of the original inequality

3. is multiplied or divided by the same *negative* number.

Note: Multiplication by 0 and division by 0 are not permitted.

If $a < b$, the **double inequality** $a < x < b$ means that $a < x$ **and** $x < b$; that is, x is between a and b. **Interval notation** is also used to describe sets defined by inequalities, as shown in Table 1.

Interval Notation

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \le x \le b$	
$[a, b)$	$a \le x < b$	
$(a, b]$	$a < x \le b$	
(a, b)	$a < x < b$	
$(-\infty, a]$	$x \le a$	
$(-\infty, a)$	$x < a$	
$[b, \infty)$	$x \ge b$	
(b, ∞)	$x > b$	

The intervals $[a, b]$, $(-\infty, a]$, and $[b, \infty)$ are closed, and the intervals (a, b), $(-\infty, a)$, and (b, ∞) are open. Note that the symbol ∞ (read infinity) is not a number. When we write $[b, \infty)$, we are simply referring to the interval that starts at b and continues indefinitely to the right. We never refer to ∞ as an endpoint, and we never write $[b, \infty]$. The interval $(-\infty, \infty)$ is the entire real number line.

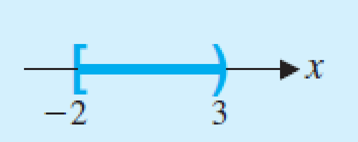
Inequalities: Example

EXAMPLE 5 Interval and Inequality Notation, and Line Graphs

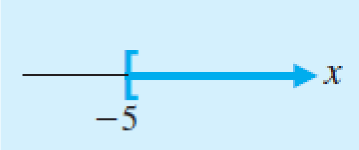
(A) Write $[-2, 3)$ as a double inequality and graph.

(B) Write $x \geq -5$ in interval notation and graph.

SOLUTION (A) $[-2, 3)$ is equivalent to $-2 \leq x < 3$.



(B) $x \geq -5$ is equivalent to $[-5, \infty)$.



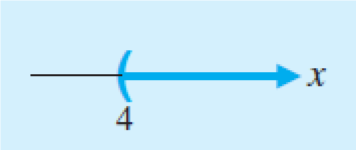
Inequalities: Example

EXAMPLE Solving a Linear Inequality Solve and graph:

$$2(2x + 3) < 6(x - 2) + 10$$

SOLUTION

$2(2x + 3) < 6(x - 2) + 10$	Remove parentheses.
$4x + 6 < 6x - 12 + 10$	Combine like terms.
$4x + 6 < 6x - 2$	Subtract $6x$ from both sides.
$-2x + 6 < -2$	Subtract 6 from both sides.
$-2x < -8$	Divide both sides by -2 and reverse the sense of the inequality.
$x > 4$ or $(4, \infty)$	



Notice that in the graph of $x > 4$, we use a parenthesis through 4, since the point 4 is not included in the graph.

Matched Problem Solve and graph: $3(x - 1) \le 5(x + 2) - 5$

Inequalities: Example

EXAMPLE 7 Solving a Double Inequality Solve and graph: $-3 < 2x + 3 \leq 9$

SOLUTION We are looking for all numbers x such that $2x + 3$ is between -3 and 9, including 9 but not -3. We proceed as before except that we try to isolate x in the middle:

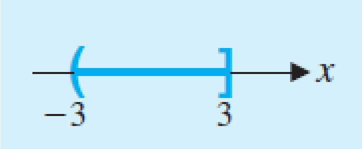
$$-3 < 2x + 3 \leq 9$$

$$-3 - 3 < 2x + 3 - 3 \leq 9 - 3$$

$$-6 < 2x \leq 6$$

$$\frac{-6}{2} < \frac{2x}{2} \leq \frac{6}{2}$$

$$-3 < x \leq 3 \quad \text{or} \quad (-3, 3]$$



Matched Problem 7 Solve and graph: $-8 \leq 3x - 5 < 7$

Applications

Applications

To realize the full potential of algebra, we must be able to translate real-world problems into mathematics. In short, we must be able to do word problems.

Here are some suggestions that will help you get started:

PROCEDURE For Solving Word Problems

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem. Often the question asked in a problem will indicate the unknown quantity that should be represented by a variable.
2. Identify other quantities in the problem (known or unknown), and whenever possible, express unknown quantities in terms of the variable you introduced in Step 1.
3. Write a verbal statement using the conditions stated in the problem and then write an equivalent mathematical statement (equation or inequality).
4. Solve the equation or inequality and answer the questions posed in the problem.
5. Check the solution(s) in the original problem.

Linear Equations: Examples

EXAMPLE Purchase Price Alex purchases a plasma TV, pays 7% state sales tax, and is charged \$65 for delivery. If Alex's total cost is \$1,668.93, what was the purchase price of the TV?

SOLUTION

Step 1 **Introduce a variable for the unknown quantity.** After reading the problem, we decide to let x represent the purchase price of the TV.

Step 2 **Identify quantities in the problem.**

Delivery charge: \$65

Sales tax: $0.07x$

Total cost: \$1,668.93

Step 3 **Write a verbal statement and an equation.**

$$\begin{array}{ccccccc} \text{Price} & + & \text{Delivery Charge} & + & \text{Sales Tax} & = & \text{Total Cost} \\ x & + & 65 & + & 0.07x & = & 1{,}668.93 \end{array}$$

Linear Equations: Examples

EXAMPLE Purchase Price Alex purchases a plasma TV, pays 7% state sales tax, and is charged \$65 for delivery. If Alex's total cost is \$1,668.93, what was the purchase price of the TV?

SOLUTION

Step 4 **Solve the equation and answer the question.**

$$x + 65 + 0.07x = 1{,}668.93 \quad \text{Combine like terms.}$$

$$1.07x + 65 = 1{,}668.93 \quad \text{Subtract 65 from both sides.}$$

$$1.07x = 1{,}603.93 \quad \text{Divide both sides by 1.07.}$$

$$x = 1{,}499$$

The price of the TV is \$1,499.

Step 5 **Check the answer in the original problem.**

$$\begin{aligned} \text{Price} &= \$1{,}499.00 \\ \text{Delivery charge} &= \$\quad 65.00 \\ \text{Tax} = 0.07 \cdot 1{,}499 &= \$\ \ 104.93 \\ \hline \text{Total} &= \$1{,}668.93 \end{aligned}$$

Matched Problem Mary paid 8.5% sales tax and a \$190 title and license fee when she bought a new car for a total of \$28,400. What is the purchase price of the car?

Linear Eqn Examples: Break-Even Point

The next example involves the important concept of **break-even analysis**, which is encountered in several places in this text. Any manufacturing company has **costs**, C, and **revenues**, R. The company will have a **loss** if $R < C$, will **break even** if $R = C$, and will have a **profit** if $R > C$. Costs involve **fixed costs**, such as plant overhead, product design, setup, and promotion, and **variable costs**, which are dependent on the number of items produced at a certain cost per item.

EXAMPLE Break-Even Analysis A multimedia company produces DVDs. Onetime fixed costs for a particular DVD are $48,000, which include costs such as filming, editing, and promotion. Variable costs amount to $12.40 per DVD and include manufacturing, packaging, and distribution costs for each DVD actually sold to a retailer. The DVD is sold to retail outlets at $17.40 each. How many DVDs must be manufactured and sold in order for the company to break even?

SOLUTION

Step 1 Let x = number of DVDs manufactured and sold.

Linear Eqn Examples: Break-Even Point

SOLUTION

Step 2

$$
\begin{aligned}
C &= \text{cost of producing } x \text{ DVDs} \\
R &= \text{revenue (return) on sales of } x \text{ DVDs} \\
\text{Fixed costs} &= \$48{,}000 \\
\text{Variable costs} &= \$12.40x \\
C &= \text{Fixed costs} + \text{variable costs} \\
&= \$48{,}000 + \$12.40x \\
R &= \$17.40x
\end{aligned}
$$

Step 3 The company breaks even if $R = C$; that is, if

$$\$17.40x = \$48{,}000 + \$12.40x$$

Step 4

$$
\begin{aligned}
17.4x &= 48{,}000 + 12.4x && \text{Subtract } 12.4x \text{ from both sides.} \\
5x &= 48{,}000 && \text{Divide both sides by 5.} \\
x &= 9{,}600
\end{aligned}
$$

The company must make and sell 9,600 DVDs to break even.

Matched Problem How many DVDs would a multimedia company have to make and sell to break even if the fixed costs are \$36,000, variable costs are \$10.40 per DVD, and the DVDs are sold to retailers for \$15.20 each?

Linear Equations Examples: Consumer Price Index (CPI)

EXAMPLE Consumer Price Index The Consumer Price Index (CPI) is a measure of the average change in prices over time from a designated reference period, which equals 100. The index is based on prices of basic consumer goods and services. Table 2 lists the CPI for several years from 1960 to 2012. What net annual salary in 2012 would have the same purchasing power as a net annual salary of $13,000 in 1960? Compute the answer to the nearest dollar. (*Source*: U.S. Bureau of Labor Statistics)

Table CPI (1982–1984 = 100)

Year	Index
1960	29.6
1973	44.4
1986	109.6
1999	156.9
2012	229.6

Linear Equations Examples: CPI

SOLUTION

Step 1 Let x = the purchasing power of an annual salary in 2012.

Step 2 Annual salary in 1960 = \$13,000

$$\text{CPI in 1960} = 29.6$$

$$\text{CPI in 2012} = 229.6$$

Step 3 The ratio of a salary in 2012 to a salary in 1960 is the same as the ratio of the CPI in 2012 to the CPI in 1960.

$$\frac{x}{13{,}000} = \frac{229.6}{29.6}$$ Multiply both sides by 13,000.

Step 4

$$x = 13{,}000 \cdot \frac{229.6}{29.6}$$

$$= \$100{,}838 \text{ per year}$$

Step 5 To check the answer, we confirm that the salary ratio agrees with the CPI ratio:

Salary Ratio	**CPI Ratio**
$\frac{100{,}838}{13{,}000} = 7.757$	$\frac{229.6}{29.6} = 7.757$

Graphs and Lines

Cartesian Coordinate System

Recall that to form a **Cartesian** or **rectangular coordinate system**, we select two real number lines—one horizontal and one vertical—and let them cross through their origins as indicated in Figure 1. Up and to the right are the usual choices for the positive directions.

These two number lines are called the **horizontal axis** and the **vertical axis**, or, together, the **coordinate axes**. The horizontal axis is usually referred to as the ***x* axis** and the vertical axis as the ***y* axis**, and each is labeled accordingly. The coordinate axes divide the plane into four parts called **quadrants**, which are numbered counterclockwise from I to IV (see Fig. 1).

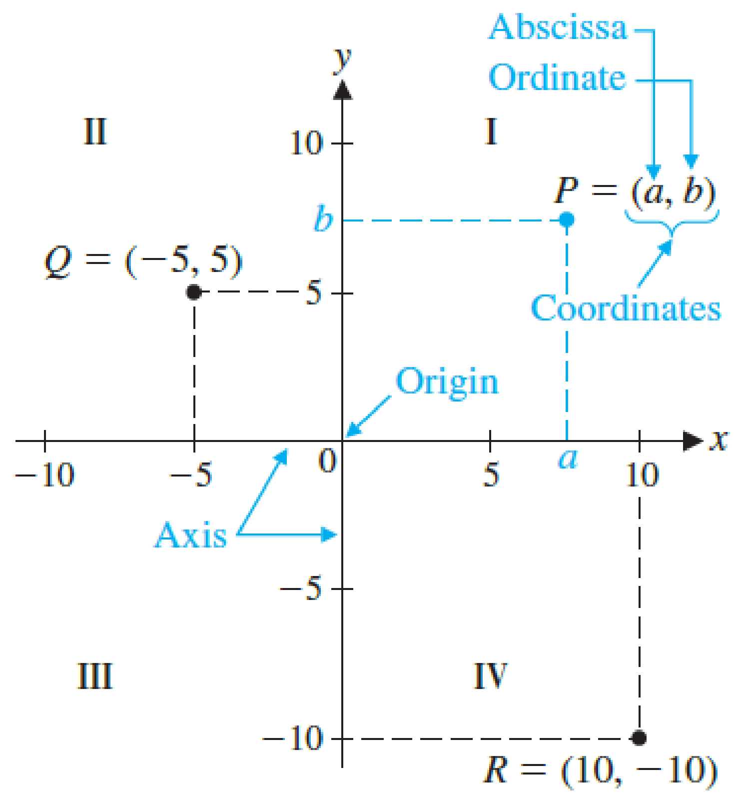


Figure 1 **The Cartesian (rectangular) coordinate system**

Cartesian Coordinate System

Now we want to assign *coordinates* to each point in the plane. Given an arbitrary point *P* in the plane, pass horizontal and vertical lines through the point (Fig. 1). The vertical line will intersect the horizontal axis at a point with coordinate *a*, and the horizontal line will intersect the vertical axis at a point with coordinate *b*. These two numbers, written as the **ordered pair** (*a*, *b*), form the **coordinates** of the point *P*.

The first coordinate, *a*, is called the **abscissa** of *P*; the second coordinate, *b*, is called the **ordinate** of *P*. The abscissa of *Q* in Figure 1 is -5, and the ordinate of *Q* is 5.

The coordinates of a point can also be referenced in terms of the axis labels. The ***x* coordinate** of *R* in Figure 1 is 10, and the ***y* coordinate** of *R* is -10. The point with coordinates (0, 0) is called the **origin**.

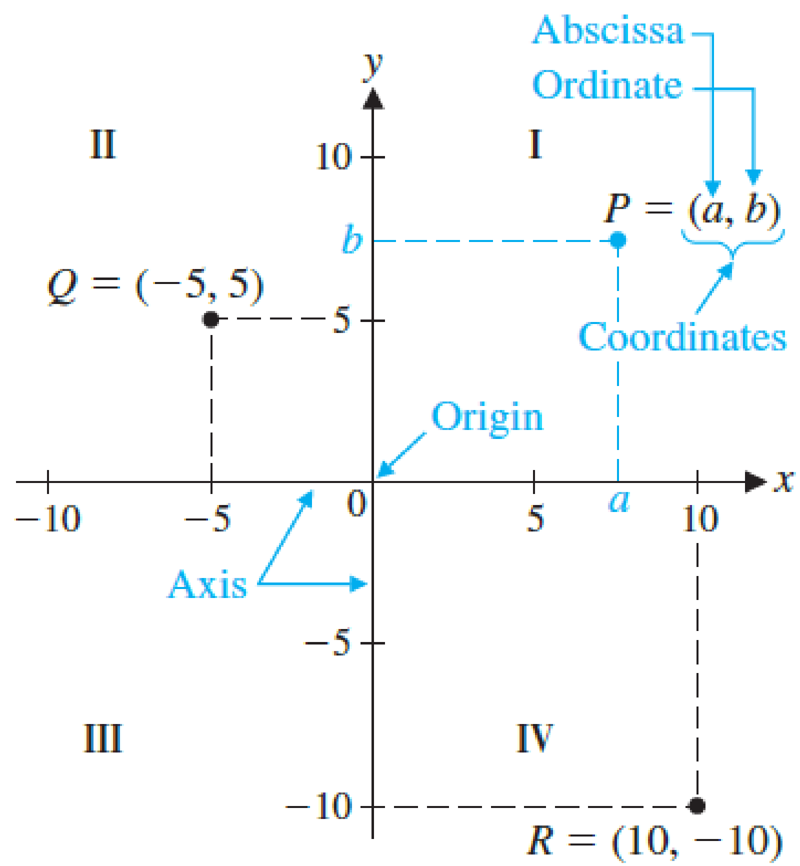


Figure 1 **The Cartesian (rectangular) coordinate system**

Cartesian Coordinate System

The procedure we have just described assigns to each point P in the plane a unique pair of real numbers (a, b). Conversely, if we are given an ordered pair of real numbers (a, b), then, reversing this procedure, we can determine a unique point P in the plane. Thus,

There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

This is often referred to as the **fundamental theorem of analytic geometry**.

Graphs of $Ax + By = C$

In Section 1.1, we called an equation of the form $ax + b = 0$ $(a \neq 0)$ a linear equation in one variable. Now we want to consider linear equations in two variables:

Cartesian Coordinate System

DEFINITION Linear Equations in Two Variables

A **linear equation in two variables** is an equation that can be written in the **standard form**

$$Ax + By = C$$

where A, B, and C are constants (A and B not both 0), and x and y are variables.

A **solution** of an equation in two variables is an ordered pair of real numbers that satisfies the equation. For example, $(4, 3)$ is a solution of $3x - 2y = 6$. The **solution set** of an equation in two variables is the set of all solutions of the equation. The **graph** of an equation is the graph of its solution set.

THEOREM 1 Graph of a Linear Equation in Two Variables

The graph of any equation of the form

$$Ax + By = C \quad (A \text{ and } B \text{ not both } 0) \tag{1}$$

is a line, and any line in a Cartesian coordinate system is the graph of an equation of this form.

Cartesian Coordinate System

$$Ax + By = C \quad \text{(}A \text{ and } B \text{ not both 0)} \qquad (1)$$

If $A \neq 0$ and $B \neq 0$, then equation (1) can be written as

$$y = -\frac{A}{B}x + \frac{C}{B} = mx + b, m \neq 0$$

If $A = 0$ and $B \neq 0$, then equation (1) can be written as

$$y = \frac{C}{B}$$

and its graph is a **horizontal line**.

If $A \neq 0$ and $B = 0$, then equation (1) can be written as

$$x = \frac{C}{A}$$

and its graph is a **vertical line**.

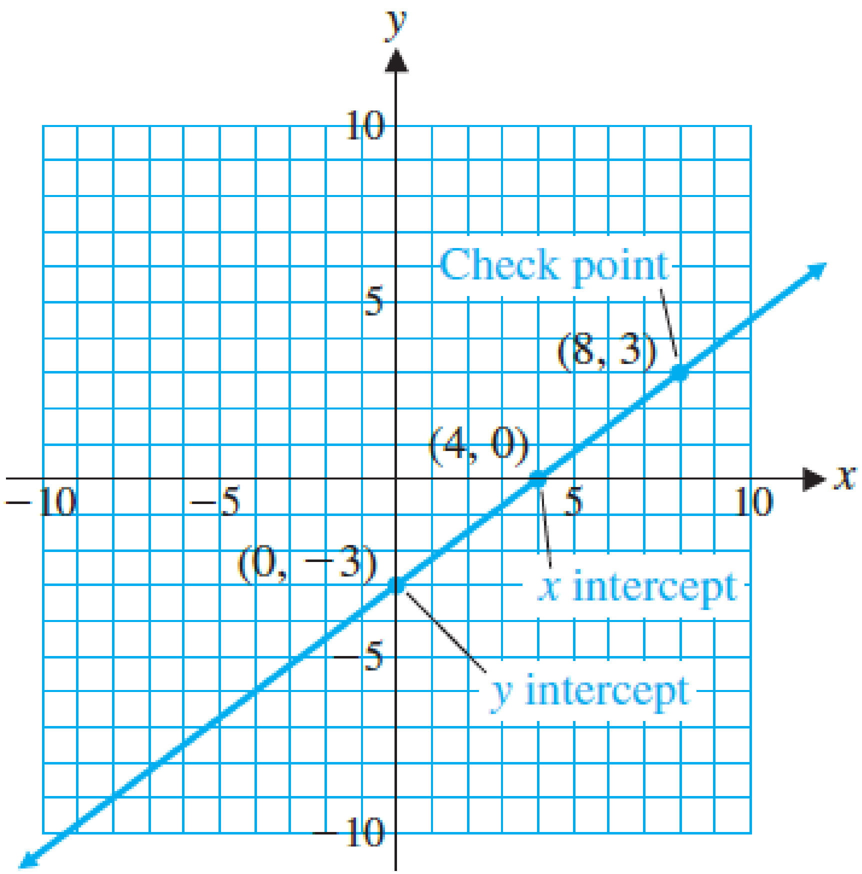
To find the y intercept, let $x = 0$ and solve for y. To find the x intercept, let $y = 0$ and solve for x. It is a good idea to find a third point as a check point.

Graphing a Line: Examples

EXAMPLE Using Intercepts to Graph a Line Graph: $3x - 4y = 12$

SOLUTION

x	y	
0	-3	y intercept
4	0	x intercept
8	3	Check point



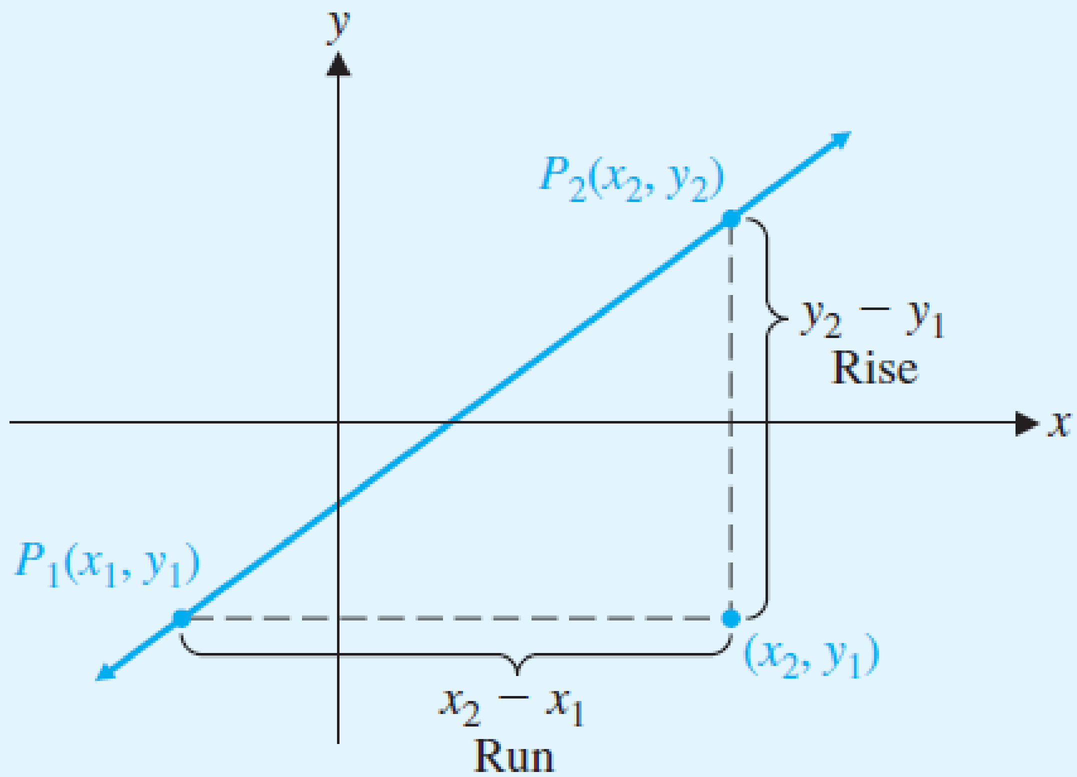
Matched Problem Graph: $4x - 3y = 12$

Slope of a Line

If a line passes through two distinct points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then its slope is given by the formula

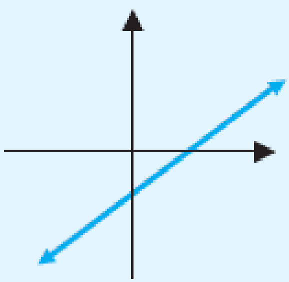
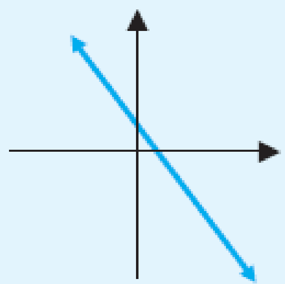
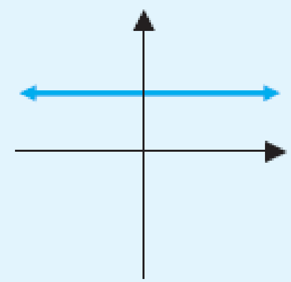
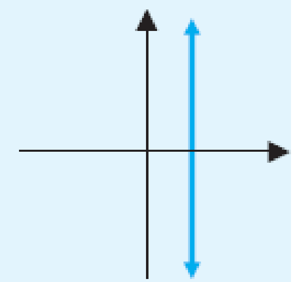
$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

$$= \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$



Geometric Interpretation of Slope

For a horizontal line, y does not change; its slope is 0. For a vertical line, x does not change; $x_1 = x_2$ so its slope is not defined. In general, the slope of a line may be positive, negative, 0, or not defined. Each case is illustrated geometrically in Table 1.

Line	Rising as x moves from left to right	Falling as x moves from left to right	Horizontal	Vertical
Slope	Positive	Negative	0	Not defined
Example				

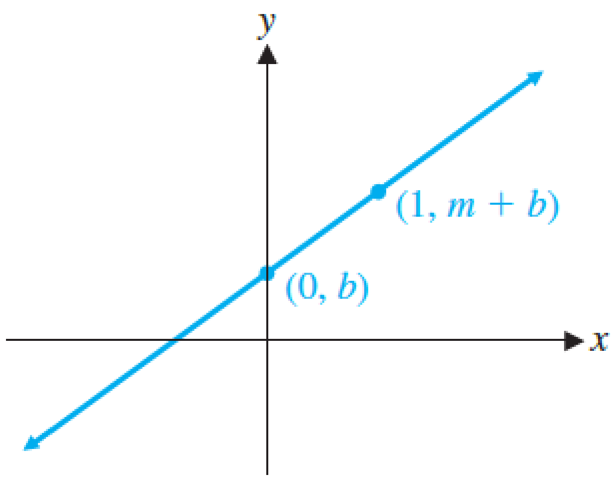
Slope-Intercept Form

The equation

$$y = mx + b \quad m = \text{slope},\ b = y \text{ intercept}$$

is called the **slope-intercept form** of an equation of a line.

So m is the slope of the line given by $y = mx + b$.



Slope-Intercept Form: Example

EXAMPLE Using the Slope-Intercept Form

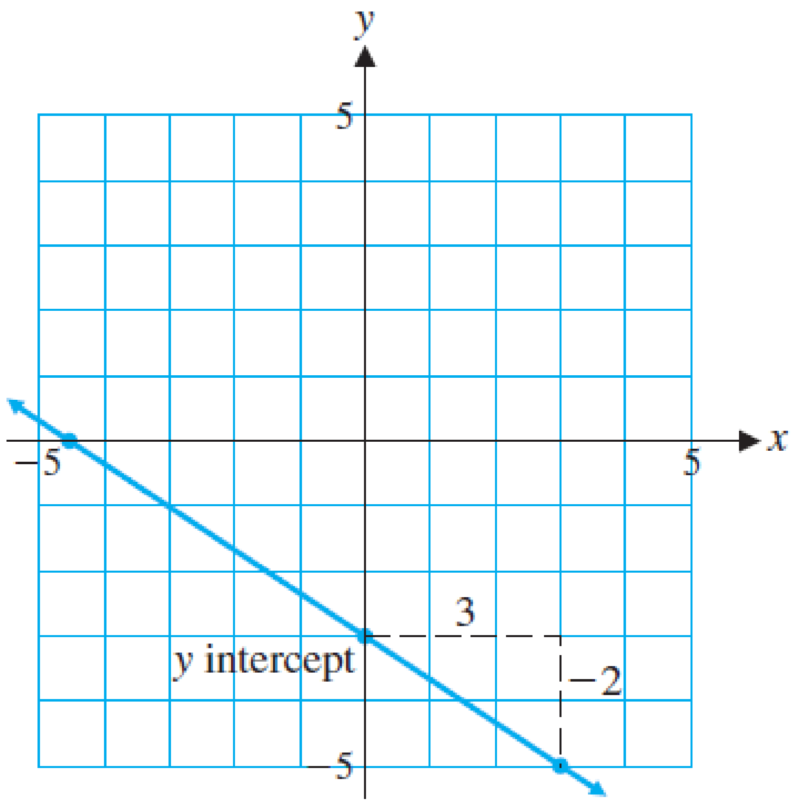
(A) Find the slope and y intercept, and graph $y = -\frac{2}{3}x - 3$.

(B) Write the equation of the line with slope $\frac{2}{3}$ and y intercept -2

SOLUTION

(A) Slope $= m = -\frac{2}{3}$

y intercept $= b = -3$



(B) $m = \frac{2}{3}$ and $b = -2$;

so, $y = \frac{2}{3}x - 2$

Point-Slope Form

Suppose that a line has slope m and passes through a fixed point (x_1, y_1). If the point (x, y) is any other point on the line (Fig. 6), then

$$\frac{y - y_1}{x - x_1} = m$$

That is,

$$y - y_1 = m(x - x_1) \tag{4}$$

DEFINITION Point-Slope Form

An equation of a line with slope m that passes through (x_1, y_1) is

$$y - y_1 = m(x - x_1) \tag{4}$$

which is called the **point-slope form** of an equation of a line.

The point-slope form is extremely useful, since it enables us to find an equation for a line if we know its slope and the coordinates of a point on the line or if we know the coordinates of two points on the line.

EXAMPLE 6 Using the Point-Slope Form

(A) Find an equation for the line that has slope $\frac{1}{2}$ and passes through $(-4, 3)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the points $(-3, 2)$ and $(-4, 5)$. Write the resulting equation in the form $y = mx + b$.

SOLUTION

(A) Use $y - y_1 = m(x - x_1)$. Let $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 3)$. Then

$$y - 3 = \tfrac{1}{2}[x - (-4)]$$

$$y - 3 = \tfrac{1}{2}(x + 4) \qquad \text{Multiply both sides by 2.}$$

$$2y - 6 = x + 4$$

$$-x + 2y = 10 \quad \text{or} \quad x - 2y = -10$$

(B) First, find the slope of the line by using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{-4 - (-3)} = \frac{3}{-1} = -3$$

Now use $y - y_1 = m(x - x_1)$ with $m = -3$ and $(x_1, y_1) = (-3, 2)$:

$$y - 2 = -3[x - (-3)]$$

$$y - 2 = -3(x + 3)$$

$$y - 2 = -3x - 9$$

$$y = -3x - 7$$

Cost Equation

EXAMPLE Cost Equation The management of a company that manufactures skateboards has fixed costs (costs at 0 output) of $300 per day and total costs of $4,300 per day at an output of 100 skateboards per day. Assume that cost C is linearly related to output x.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100; that is, the line passing through $(0, 300)$ and $(100, 4{,}300)$.

(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.

(C) Graph the cost equation from part (B) for $0 \le x \le 200$.

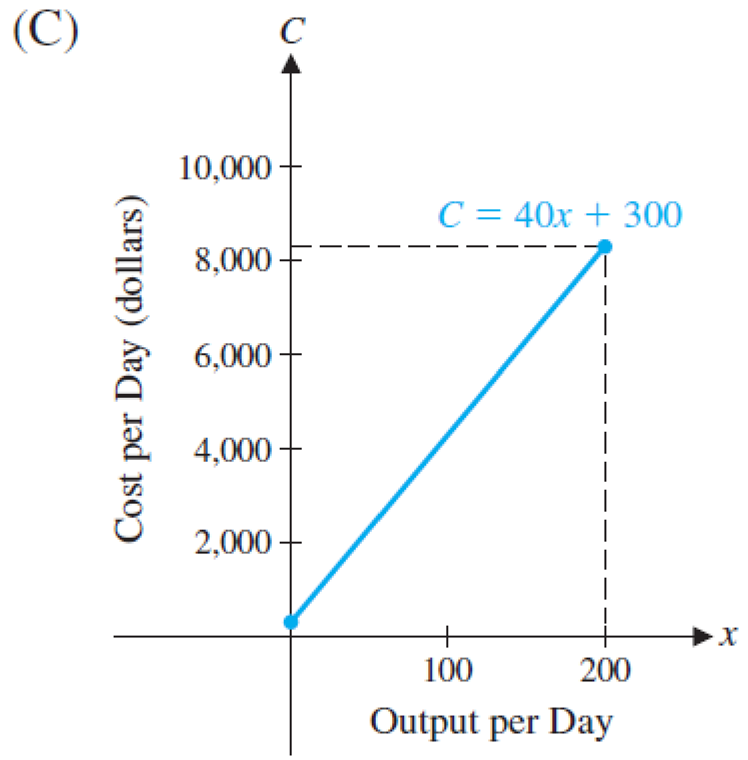
SOLUTION

(A)
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4{,}300 - 300}{100 - 0} = \frac{4{,}000}{100} = 40$$

(B) We must find an equation of the line that passes through $(0, 300)$ with slope 40. We use the slope-intercept form:

$$C = mx + b$$
$$C = 40x + 300$$

Slope-Intercept Form Examples: Cost Equation



In this example, the *fixed cost* of $300 per day covers plant cost, insurance, and so on. This cost is incurred whether or not there is any production. The *variable cost* is $40x$, which depends on the day's output. Since increasing production from x to $x + 1$ will increase the cost by $40 (from $40x + 300$ to $40x + 340$), the slope 40 can be interpreted as the **rate of change** of the cost function with respect to production x.

Slope-Intercept Form Examples: Supply and Demand

EXAMPLE Supply and Demand At a price of \$9.00 per box of oranges, the supply is 320,000 boxes and the demand is 200,000 boxes. At a price of \$8.50 per box, the supply is 270,000 boxes and the demand is 300,000 boxes.

(A) Find a price–supply equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding supply in thousands of boxes.

(B) Find a price–demand equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding demand in thousands of boxes.

(C) Graph the price–supply and price–demand equations in the same coordinate system and find their point of intersection.

Slope-Intercept Form Examples: Supply and Demand

SOLUTION

(A) To find a price–supply equation of the form $p = mx + b$, we must find two points of the form (x, p) that are on the supply line. From the given supply data, $(320, 9)$ and $(270, 8.5)$ are two such points. First, find the slope of the line:

$$m = \frac{9 - 8.5}{320 - 270} = \frac{0.5}{50} = 0.01$$

Now use the point-slope form to find the equation of the line:

$$p - p_1 = m(x - x_1) \qquad (x_1, p_1) = (320, 9)$$

$$p - 9 = 0.01(x - 320)$$

$$p - 9 = 0.01x - 3.2$$

$$p = 0.01x + 5.8 \qquad \text{Price–supply equation}$$

Slope-Intercept Form Examples: Supply and Demand

(B) From the given demand data, (200, 9) and (300, 8.5) are two points on the demand line.

$$m = \frac{8.5 - 9}{300 - 200} = \frac{-0.5}{100} = -0.005$$

$$p - p_1 = m(x - x_1) \qquad (x_1, p_1) = (200, 9)$$

$$p - 9 = -0.005(x - 200)$$

$$p - 9 = -0.005x + 1$$

$$p = -0.005x + 10 \qquad \text{Price–demand equation}$$

(C) From part (A), we plot the points (320, 9) and (270, 8.5) and then draw the line through them. We do the same with the points (200, 9) and (300, 8.5) from part (B) (Fig. 7). (Note that we restricted the axes to intervals that contain these data points.) To find the intersection point of the two lines, we equate the right-hand sides of the price–supply and price–demand equations and solve for x:

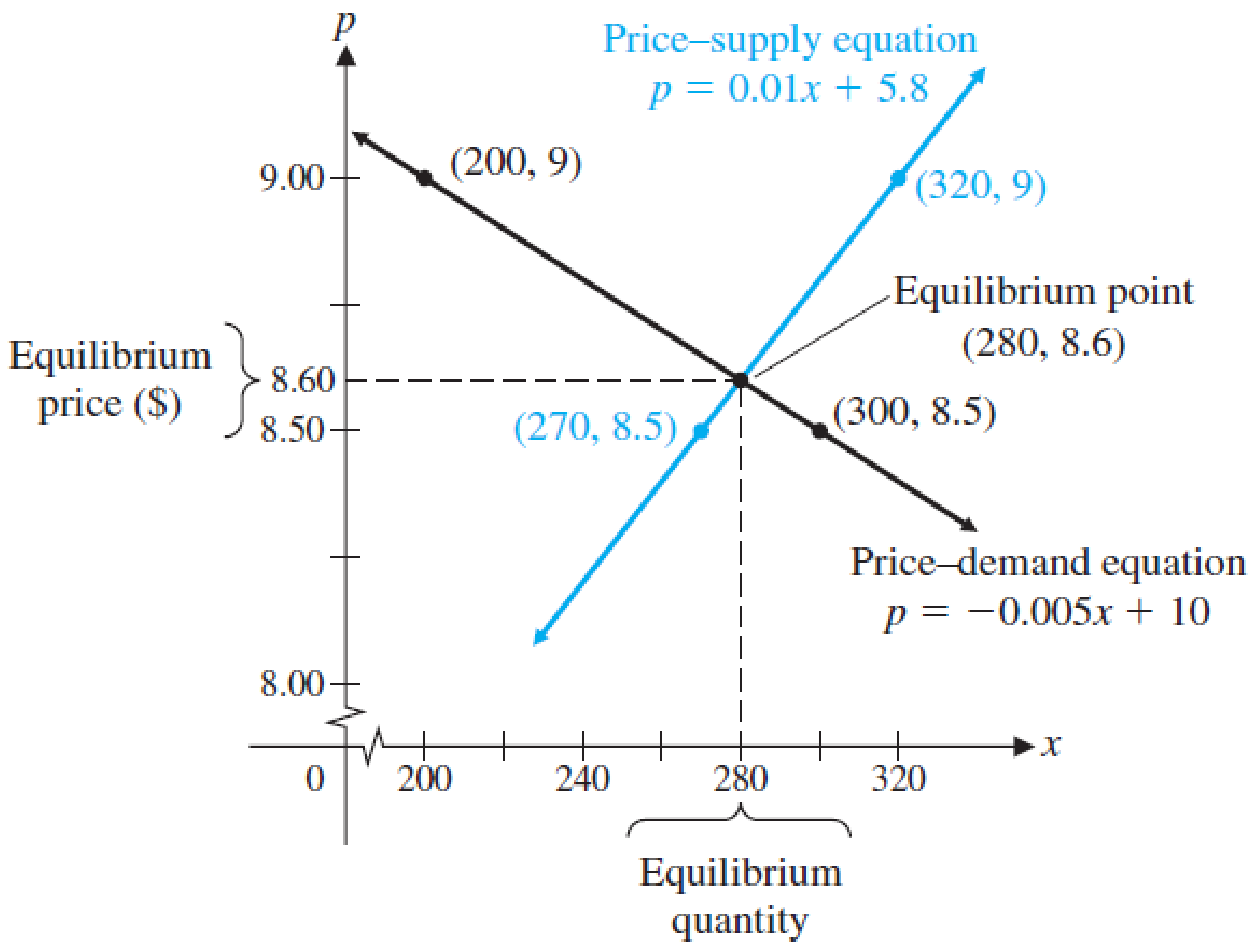
Price–supply Price–demand

$$0.01x + 5.8 = -0.005x + 10$$

$$0.015x = 4.2$$

$$x = 280$$

Slope-Intercept Form Examples: Supply and Demand



Slope-Intercept Form Examples: Supply and Demand

Matched Problem At a price of \$12.59 per box of grapefruit, the supply is 595,000 boxes and the demand is 650,000 boxes. At a price of \$13.19 per box, the supply is 695,000 boxes and the demand is 590,000 boxes. Assume that the relationship between price and supply is linear and that the relationship between price and demand is linear.

(A) Find a price–supply equation of the form $p = mx + b$.

(B) Find a price–demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.