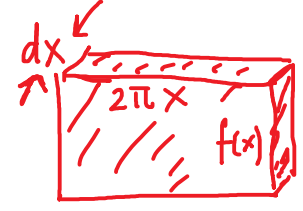
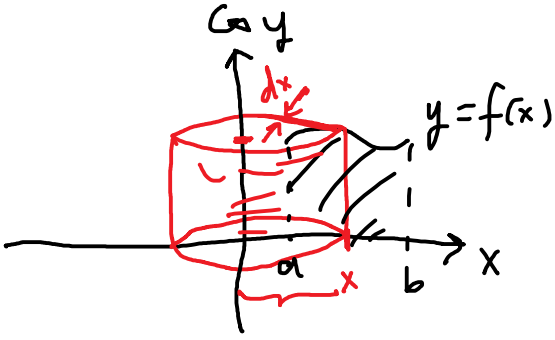


## 2. Silindirik Kabuk Yöntemi



$$A(x) = 2\pi x f(x)$$

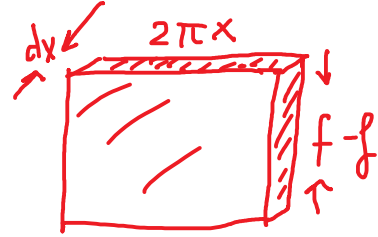
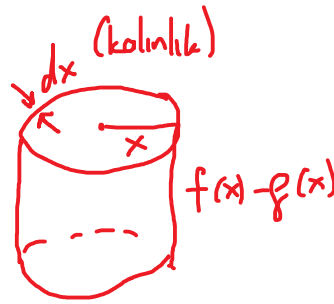
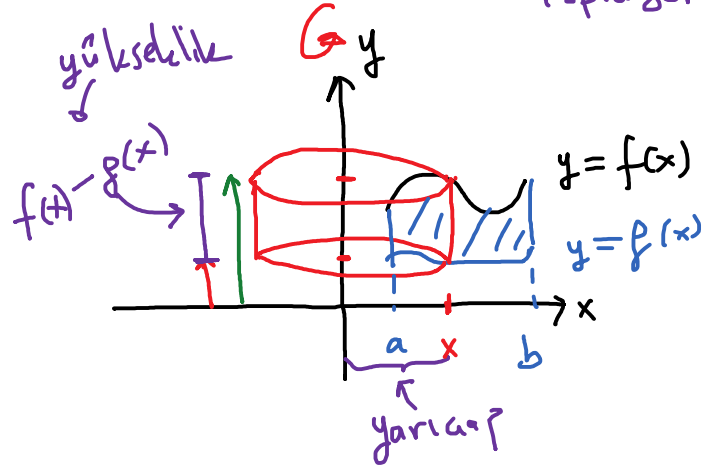
$$V = \int_a^b A(x) dx$$

← hacim

↑ silindirin yüzey alanı

← silindirin kalınlığı (çok ince)

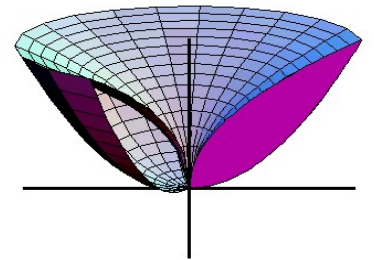
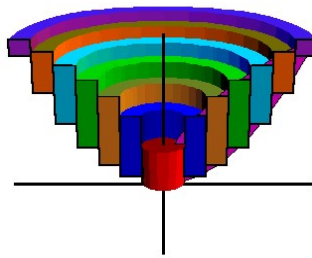
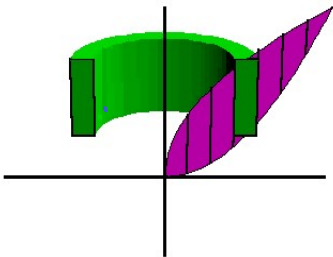
Silindirik kabukların hacmini a'dan b'ye kadar topluyor



$$A(x) = 2\pi x (f(x) - g(x))$$

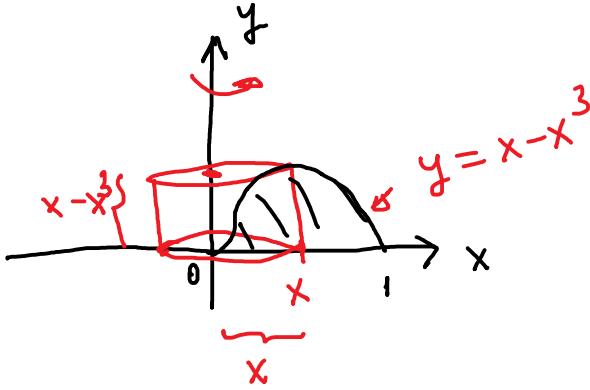
$$V = \int_a^b A(x) dx$$

Görsel Örnek:

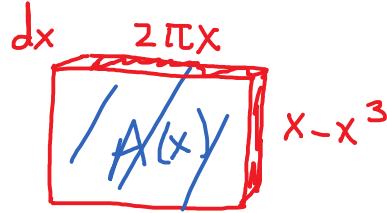
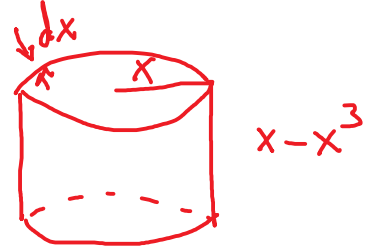


ÖRN1  $y = x - x^3$  eğrisi ile x-ekseni arasındaki bölgeyi y-ekseni çevresinde döndürerek elde edilen cismin hacmini bulunuz.

Çöz.



$$y=0 \Rightarrow x(1-x^2)=0 \\ x=0, x=1$$



$$A(x) = 2\pi x(x - x^3)$$

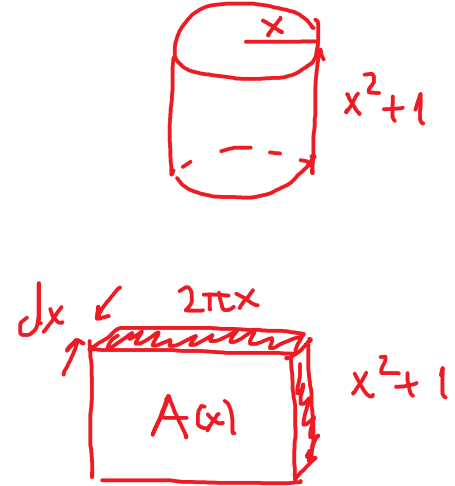
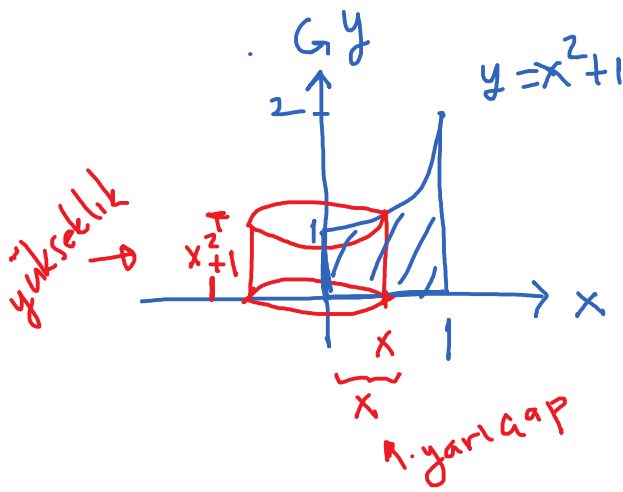
$$V = \int_0^1 A(x) dx = 2\pi \int_0^1 (x^2 - x^4) dx$$

$$= 2\pi \left( \frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1 = \frac{4\pi}{15}$$

Not: Bu soruyu disk yöntemi kullanarak cevaplamak istersek  $x$ 'i  $y$ 'nin fonksiyonu olarak yazmalıyız ancak  $y = x - x^3$  eşitliğinde  $x$ 'i yalnız bırakmak zordur Dolayısıyla bu soruda kabuk yöntemi tercih edilmelidir

ÖRNEK.  $y = x^2 + 1$ ,  $y = 0$ ,  $x = 0$  ve  $x = 1$  eğrilerinin sınırladığı bölge  $y$ -ekseni etrafında döndürüldüğünde elde edilen cismin hacmini bulunuz.

Çöz.



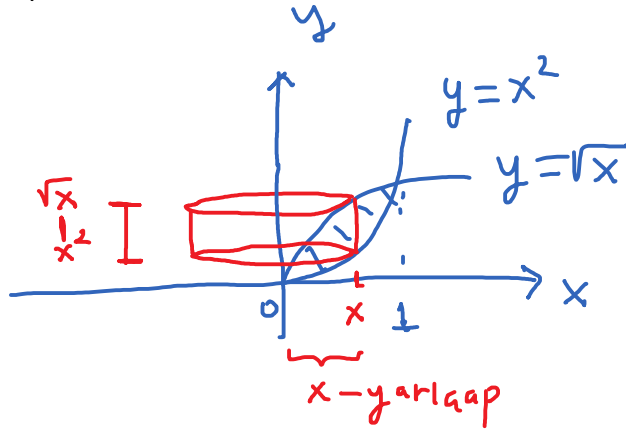
$$A(x) = 2\pi x (x^2 + 1)$$

$$V = 2\pi \int_0^1 (x^3 + x) dx$$

$$= 2\pi \left( \frac{x^4}{4} + \frac{x^2}{2} \right) \Big|_0^1 = \frac{3\pi}{2}$$

Örnek 3.  $y = \sqrt{x}$  ve  $y = x^2$  eğrilerinin sınırladığı bölgenin  $y$ -ekseni etrafında döndürülmesiyle elde edilen cismin hacmi nedir?

Çöz.



$$x^2 = \sqrt{x}$$

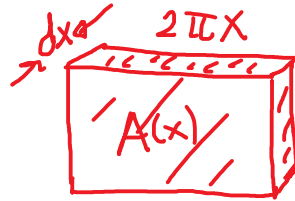
$$x^4 = x$$

$$x(x^3 - 1) = 0$$

$$x = 0, x = 1$$



$$\sqrt{x} - x^2$$



$$\sqrt{x} - x^2$$

$$A(x) = 2\pi x (\sqrt{x} - x^2)$$

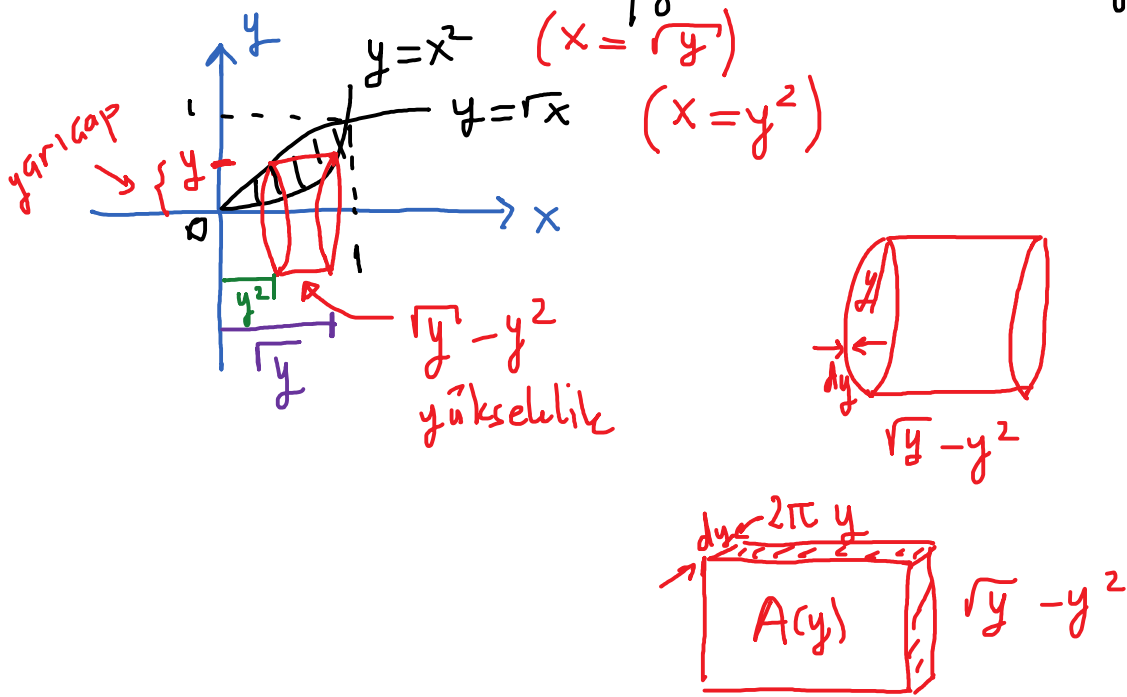
$$V = 2\pi \int_0^1 (x\sqrt{x} - x^3) dx = 2\pi \int_0^1 (x^{3/2} - x^3) dx$$

$$= 2\pi \left( \frac{x^{5/2}}{\frac{5}{2}} - \frac{x^4}{4} \right) \Big|_0^1$$

$$= 2\pi \left( \frac{2}{5} - \frac{1}{4} \right) //$$

ÖRNL.  $y = \sqrt{x}$  ve  $y = x^2$  eğrilerinin sınırladığı bölgenin x-ekseni etrafında döndürülmesiyle elde edilen cismin hacmi nedir?

GÖZ. Bu soruyu Disk yöntemi kullanarak gözlemek daha pratik olsa da silindirik kabuk yöntemi x-ekseni etrafındaki döndürmelerde nasıl uygulanır onu görelim.



$$A(y) = 2\pi y (\sqrt{y} - y^2)$$

$$V = \int_0^1 A(y) dy = 2\pi \int_0^1 (y\sqrt{y} - y^3) dy$$

$$= 2\pi \left( \frac{2}{5} y^{5/2} - \frac{y^4}{4} \right) \Big|_0^1$$

$$= 2\pi \left( \frac{2}{5} - \frac{1}{4} \right) //$$