Your Name:

Instructions: Show all of your work, and clearly indicate your answers. Use the backs of pages for the classical questions. You will need pencils/pens and erasers, nothing more. Keep all devices capable of communication turned off and out of sight.

Section I: TF questions (20 points)

Q1) T F A circle can be the graph of a function.

Q2) T F Every function has an inverse.

Q3) T F The direct substitution property can always be used to compute limits.

Q4) T F If $lim_{x\to a} f(x)$ exists then $\lim_{x\to a} f(x) = f(a)$

Q5) T F Every function is continuous on its domain.

Q7) T F $2^{\pi} = e^{2\ln\pi}$

Q8) T F $\ln x = \pi$ has a unique solution.

Q9) T F If $f(x) = x^2$ and $g(x) = x + 1$ then $(f \circ g)(x) = x^2 + 1$.

Q10) T F Every function is either an odd function or an even function.

Section II: Fill in the blank questions (20 points)

Q1) $\lim_{x\to 0} \frac{\sin(x)}{x} =$ __________

Q2) $\lim_{x\to 0} \frac{\cos(x)-1}{x} =$ __________

Q3) Let f be ________________________ on the closed interval $[a, b]$ and let $f(a) \neq f(b)$. For any number ________________ between $f(a)$ and $f(b)$ there exists a number $c \in (a, b)$ such that________________ .

Section III: Classical questions, (Show all your work)(60pts)

Q1 State the Intermediate Value Theorem.

Q2 State the Horizontal Line Test Theorem.

Q3 State the definition of continuity at a point.

Q4 State the Squeeze Theorem for limits.

Q5 Find the domain of the function $f(x) = \frac{x^2-1}{x^3\sqrt{\ln x+1}}$.

Q6 Find the limit $\lim_{x\to\frac{1}{2}} \sqrt{\frac{-4x^2+2x}{4x^2-8x+3}}$

Q7 Show that $\frac{x^4-2x-2}{x} = |x|$ has at least two roots in the interval $[-2, 2]$.

Q8 Find the limit $\lim_{x\to 0} \frac{(1+x)^3-1}{x}$

Q9 Find the limit $\lim_{x\to\infty}(x^2 - \sqrt{x^4 - x^2 - 1})$

Q10 Find the limit $\lim_{x\to 3} \sqrt{x^2 - 9}$

Q11 Show that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Q12 Find the limit $\lim\limits_{x\to 0} \dfrac{\sin(2x)}{\sin(4x)}$.

Q13 Given the function: $f(x) = \begin{cases} c - x & \text{if} \quad x \leq \pi \\ c\sin x & \text{if} \quad x > \pi \end{cases}$

Find the values of the constant c so that the function $f(x)$ is continuous.

Q14 Use polynomial long division to find the slant asymptote of $f(x) = \frac{(1-x)^3}{x^2}$

Q15 Find the horizontal asymptotes (if any) of $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 2x}$

Q16 Show that the equation $x^5 - x + 1 = 0$ has at least one real solution.

Q17 A rectangular box with a square base has volume 125. Express its total surface area A as a function of the edge length x of its base.

Q18 Write $\sin(2^{\tan(x+1)})$ as a composition of functions.

Word of the Week: "Each problem that I solved became a rule, which served afterwards to solve other problems " (*Rene Descartes*)