Instructions: Show all of your work, and clearly indicate your answers.

Q1 For what value(s) of a is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} \sin x & \text{if } x \leq c \\ ax + b & \text{if } x > c \end{cases}$$

Q2 For what value(s) of a is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} x^2 - 2x & \text{if } x \leq 6 \\ 2x + a & \text{if } x > 6 \end{cases}$$

Q3 For what value(s) of a and b is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$$

Q4 Determine c so that the function:

$$f(x) = \begin{cases} x^2 + cx + 1 & x > 1 \\ 3cx + 7 & x \leq 1 \end{cases}$$

is continuous at $x = 1$.

Q5 Consider the function:

$$f(x) = \begin{cases} 5cx - 1 & x \geq 3 \\ cx^2 - 2x + 1 & x < 3 \end{cases}$$

Determine c so that the function is continuous at 3.

Q6 Use the intermediate value theorem in order to show that the equation $x^5 - x + 1 = 0$ has at least one real solution.

Q7 Determine the value of c so that the function:

$$f(x) = \begin{cases} 3cx + 1 & x < 1 \\ 5x^2 + c & x \geq 1 \end{cases}$$

is continuous on $\mathbb{R}$.

Q8 Determine the location and type (removable, jump, infinite, or other) of all discontinuities of the function $\dfrac{x^2 - 3x + 2}{x^2 - 1}$.

Q9 Find the numbers at which the function

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

is discontinuous. At which of these points is f continuous from the right, from the left, or neither? Sketch the graph of f.

Q10 Explain why the function is discontinuous at the given point. Sketch the graph of the function.

Q11 Use the intermediate value theorem in order to show that the equation $x^3 + x^2 + 5x + 7 = 0$ has a root.

Q12 Determine the intervals on which the given function is continuous:

$$f(x) = |x - 2| + x$$

Q13 Determine the intervals on which the given function is continuous:

$$f(x) = \sqrt{-x^2}$$

Q14 Determine the intervals on which the given function is continuous:

$$f(x) = \frac{1 + \cos x}{3 + \sin x}$$

where $x \in [0, 2\pi]$.

Q15 Determine the intervals on which the given function is continuous:

$$f(x) = \frac{x + 1}{x(x - 1)(x^2 - 2)}$$

Q16 Find the points of discontinuity of the function

$$f(x)=\frac{x+4}{x^2-x-2}$$

Q17 Determine the intervals on which the given function is continuous:

$$f(x)=\begin{cases}\frac{x^2+3x-10}{x-2} & \text{if } x\neq 2\\ 10 & \text{if } x=2\end{cases}$$

Q18 Show that there exists a real number x whose cosine is twice that number.

Q19 Find the limit if it exists

$$\lim_{x\to a}\frac{\cos x-\cos a}{\sin x-\sin a}$$

Q20 Find the limit if it exists

$$\lim_{x\to a}\frac{\sin(x-a)}{x^2-a^2}$$

Q21 Find the limit if it exists

$$\lim_{x\to \pi/4}\frac{\cos 2x}{\cos x-\sin x}$$

Q22 Find the limit if it exists

$$\lim_{x\to 0}\frac{x}{x+\sin x}$$

Q23 Find the limit if it exists

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x}}$$

Q24 Find the limit if it exists

$$\lim_{x\to 0}\frac{\tan x-x}{\sin x}$$

Q25 Find the limit if it exists

$$\lim_{x\to 0}\left(1+2^{\frac{1}{x}}\right)$$

Q26 Determine all **horizontal or slant asymptotes** of the function $f\left(x\right)=\dfrac{1-2x}{\sqrt{3x^2+1}}$

Q27 Determine all **horizontal or slant asymptotes** of the function $f\left(x\right)=\dfrac{2x^3}{x^2+1}$

Q28 Find all discontinuous points of $f\left(x\right)$

$$f\left(x\right)=\frac{x^2+1}{x^2+x-6}$$

Q29 Find all discontinuous points of $f\left(x\right)$

$$f\left(x\right)=x\csc x$$

Q30 Find all discontinuous points of $f\left(x\right)$

$$f\left(x\right)=\begin{cases}2x & \text{if} \quad x<0\\ \sin x & \text{if} \quad x=0\\ x-\pi & \text{if} \quad x>0\end{cases}$$

Q31 Find a slant asymptote of $f\left(x\right)=\dfrac{x^3+1}{x^2+2}$.

Q32 Find the horizontal asymptote of the graph of f if it exists.

$$a.\ f\left(x\right)=\frac{2x^2-x+1}{1-3x^2}\qquad b.\ f\left(x\right)=\frac{x}{3-x^2}\qquad c.\ f\left(x\right)=\frac{x^3+1}{x^2+2}$$

Word of the Week: "We must know, we will know" (*David Hilbert*)