Your Name:

Instructions: You will need pencils/pens and erasers, nothing more. Keep all devices capable of communication turned off and out of sight.

Fill in the blank questions (100 points)

Q1) The ______________________ between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Q2) Two (nonvertical) lines are ______________________ if and only if they have the same ______________________ .

Q3) A **function** f is a ______________________ that assigns to each element x in a set A ______________________ $f(x)$, in a set B.

Q4) [Poincaré Conjecture(original form)(1904)] Every simply connected, closed 3-manifold is______________________ to the 3-sphere.

Q5) ______________________ is a measure of how quickly a tangent line turns on a curve.

Q6) The ______________________ of a vector $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, written as $|\mathbf{x}|$, is the square root of the ______________________ of the vector with itself,

$$|\mathbf{x}| = \langle \mathbf{x}, \mathbf{x} \rangle^{\frac{1}{2}} = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Q7) A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is ______________________ at $p \in \mathbb{R}^n$ if for each $\epsilon > 0$, there exists $\delta > 0$ such that

$$f(B_\delta(p) \subset B_\epsilon(f(p))$$

Q8) Let $U \subset \mathbb{R}^n$ be open. $f : U \to \mathbb{R}^m$ is ______________________ if and only if whenever $V \subset \mathbb{R}^m$ is open, $f^{-1}(V)$ is also open.

Q9) A function $f : X \to Y$ is a ______________________ if it is a ***bijection***, and both f and its inverse f^{-1} are ______________________ functions.

Q10) Two topological spaces are ***equivalent*** (as topological spaces) if there is a ______________________ from one topological space onto the other.

Q11) Let $\alpha : [a, b] \to \mathbb{R}^3$ be a ***regular*** curve. Then the ______________________ of α is given by

$$L = \int_a^b \left| \frac{d\alpha}{dt} \right| dt.$$

Q12) If f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)dt$, then

$$F'(x) = \text{______________________}$$

Q13) Suppose that f, g are differentiable on $[a, b]$ with f', g' integrable on $[a, b]$. Then

$$\int_a^b f'(x)g(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f(x)g'(x)dx.$$

Q14) Let ϕ be continuously differentiable on a closed, bounded interval $[a, b]$. If f is continuous on $\phi([a, b])$ then

$$\int_{\phi(a)}^{\phi(b)} f(t)dt = \int_a^b f(\phi(x))\phi'(x)dx.$$

Q15) Let f be continuous function on $[0, 1]$. Then evaluate

$$\int_{-1}^{1} x f(x^2)dx.$$

Q16) An n-manifold is a space that looks locally like Euclidean space $\mathbb{R}^n$.