Green's/ Divergence/ Stokes Integrals

Question Verify the formula div(**curl F**) = 0 where

$$\mathbf{F} = M(x,y,z)\mathbf{i} + N(x,y,z)\mathbf{j} + P(x,y,z)\mathbf{k}$$

.

Question1 Use Green's theorem to evaluate the line integral

$$\int_C (x+y)\,dx + (3y+y^2-x^2)\,dy$$

where C is the triangle with vertices $(0,0)$, $(2,0)$ and $(2,4)$ oriented in a counterclockwise direction.

Question2 Verify that the vector field

$$\mathbf{F}(x,y,z) = (y+2z)\mathbf{i} + (x-3z)\mathbf{j} + (2x-3y)\mathbf{k}$$

is conservative.

Question3 Show that Area of

$$R = \frac{1}{2}\int_C x\,dy - y\,dx$$

.

Question4 Find the area enclosed by the ellipse

$$\mathbf{r}(t) = (a\cos t)\mathbf{i} + (b\sin t)\mathbf{j}$$

for $0 \le t \le 2\pi$, where a and b are constants.

Question5 Evaluate the line integral

$$\int_C xy\,dx + x^2\,dy$$

where C is the parabolic arc given by $x = y^2$ from $(4,-2)$ to $(9,3)$.

Question6 Evaluate the line integral

$$\int_C y\,dx + x^2\,dy$$

where C is the parabolic arc given by $y = 4x - x^2$ from $(4,0)$ to $(1,3)$.

Question7 Verify the formula curl$(\nabla f) = \mathbf{0}$ where $f(x,y,z)$ has continuous second partial derivatives. Why is it important for f to have continuous second partial derivatives?

Question8 Is the vector field

$$\mathbf{F}(x,y,z) = (2x+y^2)\mathbf{i} + (2xy+e^z)\mathbf{j} + ye^z\mathbf{k}$$

conservative?

Question9 Evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ along the curve

$$\mathbf{r}(t) = t\mathbf{i} + (1-t)\mathbf{j} + t^2\mathbf{k}$$

where $0 \le t \le 2$ and

$$\mathbf{F}(x,y,z) = (2x+y^2)\mathbf{i} + (2xy+e^z)\mathbf{j} + ye^z\mathbf{k}$$

.

Question10 Use Green's theorem to evaluate the line integral

$$\int_C (x-4y)\,dx + (3y+e^{y^5}+x^2)\,dy$$

where C is the triangle with vertices $(0,0)$, $(2,0)$ and $(0,4)$ oriented in a counterclockwise direction. How would the result change if C had a clockwise orientation?

Question11 Suppose C is simple closed curve with counterclockwise orientation enclosing a region R satisfying the hypothesis of Green's theorem. Show that the area of R is $\int_C x\,dy$.

Question12 Let S be the surface of the cube bounded by the planes $x=0$, $x=2$, $y=0$, $y=4$, $z=0$, $z=3$, and let S be oriented with outward unit normal. Use the divergence theorem to find

$$\int\int_S \mathbf{F}\cdot\mathbf{N}\,dS$$

where

$$\mathbf{F}(x,y,z) = (x + ze^y \sin(yz))\mathbf{i} + (z\sin(e^z)\cos z + y^2)\mathbf{j} - z\mathbf{k}$$

.

Question13 Find the volume of an ice cream cone bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.

Question14 Let S be the surface $z = x^2 + y^2$ where $0 \le z \le 4$. Set up the integral to find the flux of the vector field

$$\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

through the surface S oriented with upward unit normal.

Question15 Calculate the flux of the vector field

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$$

across the surface S given by

$$z = \sqrt{1 - x^2 - y^2},$$

where (x,y) ranges over the unit disk.

Question 16 Evaluate $\int_C x + y^2 - z\,ds$ where C is the straight-line segment from $(0,1,1)$ to $(1,0,2)$.

Question 17 Compute the work done by the field $F = \langle 2x+3y^3, 9xy^2+2y\rangle$ as a particle is moved from $(1,0)$ to $(2,0)$ along the path

$$r(t) = \langle t^{17} + 1, t^3 - t^2\rangle$$

.

Question 18 Evaluate the line integral $\int_C (2xy^2 - 5\cos x)\,dx + (-3x^2 - 7e^y)\,dy$, where C is the boundary of the rectangle with vertices $(0,1)$, $(2,1)$, $(2,2)$, and $(0,2)$, with positive (counterclockwise) orientation.

Question 19 A simply connected region R in the plane satisfies the following

$$\iint_R 1\,dA = 5, \quad \iint_R x\,dA = 5, \quad \iint_R y\,dA = 5.$$

What is the value of the line integral

$$\int_C \left(x^2 + xy + 3y\right)\,dx + \left(\arctan(y^2) + 3x^2 + 2xy + x\right)\,dy$$

, where C is the boundary of R oriented in a counterclockwise direction.

Question 20 Evaluate $\int_C (x^2+y^2)\,dx + 2xy\,dy$, where:

(a) $C : x = t\ , \quad y = 2t\ , \quad 0 \le t \le 1$

(b) $C : x = t\ , \quad y = 2t^2\ , \quad 0 \le t \le 1$

Question 21 Evaluate $\int_C (x^2+y^2)\,dx + 2xy\,dy$, where C is the polygonal path from $(0,0)$ to $(0,2)$ to $(1,2)$.

Question 22 Evaluate $\oint_C (x^2+y^2)\,dx + 2xy\,dy$, where C is the boundary (traversed counterclockwise) of the region

$$R = \{\,(x,y) : 0 \le x \le 1,\ 2x^2 \le y \le 2x\,\}$$

.

Question 23 A *torus* T is a surface obtained by revolving a circle of radius a in the yz-plane around the z-axis, where the circle's center is at a distance b from the z-axis $(0 < a < b)$. Find the surface area of T.

Question 24 Find the surface area of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$

Question 25 Evaluate

$$I = \int\int_S x^2y^2z^2\,dS$$

over the curved surface of the cone $x^2 + y^2 = z^2$ which lies between $z = 0$ and $z = 1$.

Question 26 Find the surface area of the paraboloid given by

$$z = 4 - x^2 - y^2$$

for $z \geq 0$.

Question 27

1. Sketch the surface S given by the equation

$$x + 2y + 3z = 6$$

in the first octant.

2. Find the surface area of S.

Question 28 Evaluate

$$\int\int_S (3x\overrightarrow{i} + 2y\overrightarrow{j})dS$$

where S is the sphere

$$x^2 + y^2 + z^2 = 9.$$

Question 29 Evaluate

$$\int\int_S ydS$$

where S is the surface $z = x + y^2$, $0 \leq x \leq 1$, $0 \leq y \leq 2$.

Question 30 Let S be the boundary of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$, with outward orientation.

1. Find the surface area of S. Note that the surface S consists of a portion of the paraboloid $z = x^2 + y^2$ and a portion of the plane $z = 1$.

2. Use the Divergence Theorem to calculate the surface integral $\int\int_S F \cdot dS$, where $F = (x + y^2z^2)\mathbf{i} + (y + z^2x^2)\mathbf{j} + (z + x^2y^2)\mathbf{k}$.