Sample Test I

S0

Determine which of the following series converge. Justify your answer.

(a) $\sum_{n=2}^{\infty} \frac{1}{[n+(-1)^n]^2}$

(b) $\sum[\sqrt{n+1}-\sqrt{n}]$

(c) $\sum \frac{n!}{n^n}$

(d) $\sum_{n=2}^{\infty} \frac{\log n}{n}$

(e) $\sum_{n=4}^{\infty} \frac{1}{n(\log n)(\log\log n)}$

S1(Not included)

Find the radius of convergence and determine the exact interval of convergence:

$$\sum \left(\frac{n^3}{3^n}\right) x^n$$

S2

1. Let $\langle a_n\rangle_{n\in\mathbb{N}}$ be a sequence. Define what it means to say $\sum_{n=0}^{\infty} a_n$ converges.

2. Determine whether the following statement is true: For every sequence $\langle a_n\rangle_{n\in\mathbb{N}}$ from $\mathbb{R}$, $\sum_{n=0}^{\infty} a_n$ converges if and only if $\lim_{n\to\infty} a_n = 0$. [If true, then give a proof. If false, then give a counterexample.]

S3(Not included)

Compute the Taylor series about 0, for the function $f(x) = e^x$.

S4(Not included)

Using the ratio or root test, determine the convergence or divergence of the following series

1. $\sum_{n=1}^{\infty} \frac{n!}{2^{e^n}}$
2. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
3. $\sum_{n=1}^{\infty} \frac{n^3+2^n}{e^n}$

S5(Not included)

Find the Taylor expansion of $\ln(x)$ at $x = 2$. Specify its radius and interval of convergence.

S6(Not included)

Find a power series representation for the following function $f(x)$. Also, find the interval of convergence.

$$f(x) = \frac{x}{2+x}$$

.

S7(Not included)

Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$$

Q8

Let $f(x) = \int_2^x (t^3 - t^2 - 4)\, dt$ then find $f'(t)$.

Q9

Evaluate $\int \cos^3 x\, dx$

Q10

Find the volume of the solid that is obtained when the region under the curve $y = x$ over the interval $[1, 2]$ is revolved about the $x-$axis.

Q11

Compute the length of the arc of the semicubical parabola $y^2 = x^3$ between the points $(0, 0)$ and $(4, 8)$.

Q12

Evaluate

$$\int \frac{1}{x(x-2)} dx$$

Q13

Evaluate.

$$\int_0^\infty xe^{-x}\, \mathrm{d}x$$

Q14

Show that the surface area of a unit sphere is 4π.

Q15

Evaluate $\int \sqrt{a^2 - x^2} dx$.

Q16

Show that

$$\int_{-a}^{a} \sqrt{a^2 - x^2} dx = \frac{\pi a^2}{2}.$$

Q17

Evaluate $\int \cos^5(x) dx$.

Q18

Evaluate $\int_0^{\pi/4} \sqrt{1 + \cos(4x)} dx$.

Q19

Evaluate $\int \frac{3x - 1}{x^2 - x - 6} dx$.

Q20

Determine whether or not $\int_1^\infty \frac{1}{x + e^x} dx$ converges or diverges.

Q21

Evaluate $\int_1^4 \frac{3x + 1}{(x - 5)(x + 3)} dx$

Q22

Evaluate $\int e^{2x} \sqrt{1 - e^{2x}} dx$.

Q23(Not included)

Determine the radius of convergence of: $\sum_{n=1}^{+\infty} n! x^n$.

Q24(Not included)

Let $f(x) = \cos(5x^2)$. Find $f^{(100)}(0)$.

Q25

Compute the sum of the series: $\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}$

Q26

Compute $\int \cos(\sqrt{x})dx$

Q27

Determine whether or not $\sum_{n=1}^{\infty} \frac{n+1}{n^3}$ converges or diverges. Give your reasoning.

Q28

Determine whether or not $\sum_{n=2}^{\infty} \frac{(n+1)^2}{n^3 \ln n}$ converges or diverges. Give your reasoning.

Q29

Determine whether or not $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+21}$ converges or diverges. Give your reasoning.

Q30

If $f(0) = g(0) = 0$, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

Q31

Evaluate the integral $\int e^{-\theta} \cos 3\theta \, d\theta$.

Q32

Determine whether the following integral is convergent or divergent. Evaluate it if it is convergent.

$$\int_1^9 \frac{1}{\sqrt[3]{x-9}}\,dx$$

Q33

A series $\sum a_n$ is defined by the equations

$$a_1 = 1, \quad a_{n+1} = \frac{2 + \cos n}{\sqrt{n}} a_n$$

Determine whether $\sum a_n$ converges or diverges.

Q34

Use substitution to compute

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx.$$

Q35

Use integration by parts to compute

$$\int \cos x \ln(\sin x)\,dx.$$

Q36

Compute the trigonometric integral

$$\int \sin^2\theta \cos^2\theta \, d\theta.$$

Q37

Evaluate the definite integral.
$\int_1^2 x\sqrt{x-1}dx$

Q38

Evaluate the integral by interpreting it in terms of area $\int_{-2}^{2} \sqrt{4-x^2}dx$

Q39(Not included)

Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(4-2x)^n}{3n-1}$.

Q40

Determine whether $\sum_{i=1}^{\infty} \sin(1/n)$ converges or diverges.

Q41

Evaluate $\int \frac{\ln(\tan(x))}{\sin(x)\cos(x)}dx$.

Q42

Evaluate $\int \sqrt{4-x^2}dx$.

Q43

Evaluate $\int \tan(x)^3 dx$.

Q44

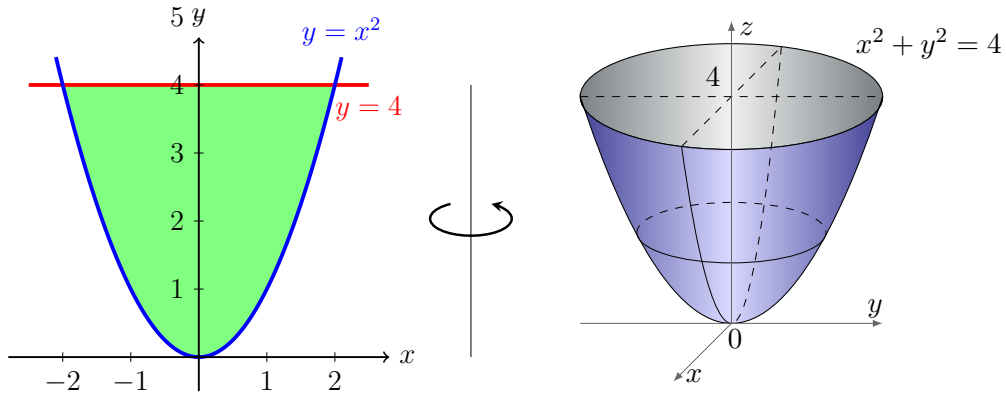
Evaluate $\int_0^{\pi/4} \frac{\tan(x)+1}{\cos(x)^2}dx$.

Q45

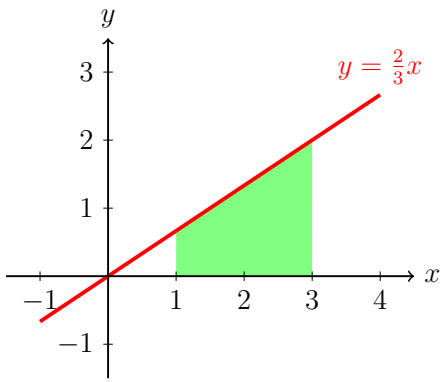
Evaluate the integral

$$\int_0^{\pi} \cos^4 x \sin x dx$$

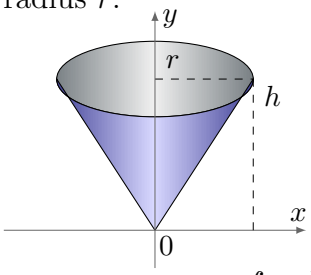
Q46 Find the surface area of the solid generated when the region between the graphs of the functions $y = x^2$ and $y = 4$ over the interval [-2, 2] is revolved about the y-axis.



Q212 Find the volume of the solid generated when the region between the graphs of the functions $y = \frac{2}{3}x$, $x = 1$ and $x = 3$ and the x-axis is revolved about the x-axis.



Q208 Find the surface area S of a right circular cone of height h and radius r.



Q250 Compute $\int \frac{dx}{x^3 - x}$.

(A) $\ln(\frac{\sqrt{x^2-1}}{|x|}) + c$

(B) $A - B - C + D$

(C) $A - B + C + D$

(D) $A - B - C + D$

(E) None

Q251 Compute $\int_0^1 \frac{dx}{e^x + e^{-x}} dx$

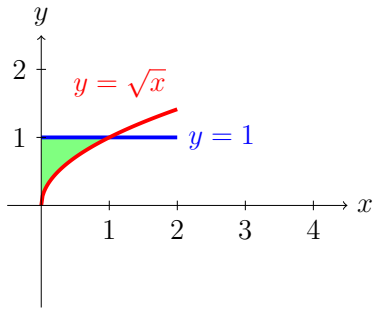
(A) $A - B - C + D$

(B) $A - B + C + D$

(C) $A - B - C + D$

(D) None

Q252 The region bounded by $y = \sqrt{x}$, $y = 1$ and $x = 0$ is rotated about the y-axis to form a solid. Write an in integral that uses disk method to compute the volume.



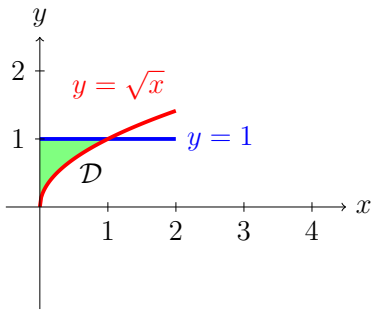
(A) e

(B) $e + 1$

(C) e^2

(D) None

Q253 The region bounded by $y = \sqrt{x}$, $y = 1$ and $x = 0$ is rotated about the y-axis to form a solid. Write an in integral that uses shell method to compute the volume.



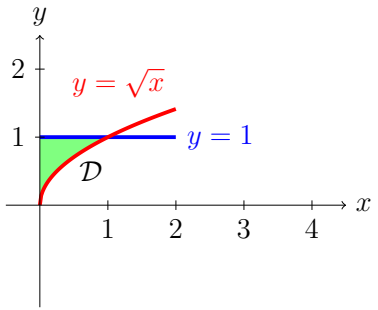
(A) e

(B) $e + 1$

(C) e^2

(D) None

Q254 The region bounded by $y = \sqrt{x}$, $y = 1$ and $x = 0$ is rotated about the y-axis to form a solid. Write an in integral that uses disk or shell method to compute the volume.



(A) e

(B) e

(C) e

(D) None

Q255 The improper integral $\int_e^\infty \frac{dx}{x \ln x} dx$

(A) diverges
(B) converges and equals 2
(C) converges and equals $\sqrt{2}$
(D) $A-B-C+D$
(E) None

Q256 If we want to decompose $\frac{x^2+4x+5}{(x+1)(x^2+2x+5)^2}$ into partial fractions, we should look for an expression of the form:

Q257 Let f be differentiable function. Which of the expression is the same as the integral $\int \frac{1}{x^2} f(x)dx$

(A) $\frac{-f(x)}{x} + \int \frac{f'(x)}{x}dx$
(B) a
(C) a
(D) a
(E) None

Q258 If $a > 0$ is a constant, then $\int_0^2 x^{a-1}e^{x^a}dx$ equals:

(A) $\frac{1}{a}(e^{2^a}-1)$
(B) a
(C) a
(D) a
(E) None

Q259 The integral $\int \frac{\cos x}{1+sin^2 x}dx$ equals:

(A) a
(B) a
(C) a
(D) None

Q260 Which of the following gives the value of the integral $\int_5^\infty \frac{dx}{(x-3)^2}dx$

(A) 1/2
(B) a
(C) a
(D) a
(E) None

Q261 Which of the following gives the value of the integral $\int_0^1 \frac{dx}{x-1}dx$

(A) *diverges*
(B) -12
(C) 1
(D) $A-B-C+D$
(E) None

Q262 Which of the following gives the value of the area under the curve $y = \frac{1}{x^2+1}$ in the first quadrant?

Q263 To which of the following numbers does the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdot$ converge?

(A) *diverges*
(B) -12
(C) 1
(D) $A-B-C+D$
(E) None

Q264 Evaluate the series $\sum_{n=1}^{\infty} 2^{4-3n}$

(A) 128/7
(B) $A-B-C+D$
(C) $A-B-C+D$
(D) $A-B-C+D$
(E) None

Q265 Consider the series $\sum_{n=1}^{\infty} \frac{1}{2^n+5n-2}$. Using the comparison test with the series leads to the following result. There is only one correct answer.

(A) The series converges
(B) The test is inconlusive
(C) The test diverges
(D) The test is not applicable
(E) None

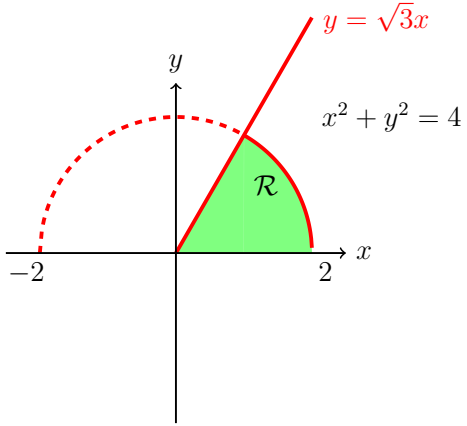
Q265 Which of the following integrals converge?

1. $\int_- 1^1 \frac{1}{1+x^2}dx$
2. $\int_1^\infty e^{-x}dx$
3. $\int_0^1 \frac{1}{x^{3/2}}dx$

(A) only 1 | (C) 1 and 2 | (E) None

(B) only 2 | (D) 2 and 3

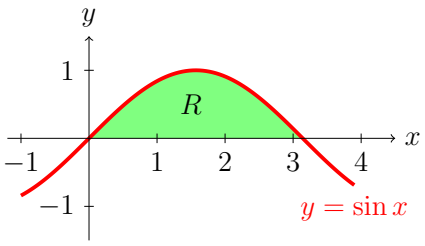
Q266 Let R be region enclosed by the curves $x^2 + x^2 = 4$, $y = \sqrt{3}x$, and the x-axis. Rotate R about the y-axis and find the resulting volume.



(A) $\frac{8\pi}{3}\sqrt{3}$ | (C) $A-B+C+D$ | (E) None

(B) $A-B-C+D$ | (D) $A-B-C+D$

Q267 Let R be the region in the first quadrant that is under the curve $y = \sin x$ on the interval $[0, \pi]$. Find the volume of the solid obtained by rotating R about the $y-$axis.



(A) $2\pi^2$ | (C) $2\sqrt{2} - 2$ | (E) None

(B) $\sqrt{2}$ | (D) $\sqrt{2} + 1$

Q268 What conclusion can we draw about the convergence of if we use the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(A) The series converges absolutely. | (B) The series diverges. | (C) The test is inconclusive. | (D) None

Q269 Which of the following integrals represents the arclength of

$$y = 3sin(x^2)$$

over $[0, 2]$?

(A) $2\pi \int_0^2 \sqrt{1 + \sin x^2} dx$ | (C) $2\pi \int_0^2 \sqrt{1 + 36x^2 \sin^2(x^2)} dx$

(B) $2\pi \int_0^2 \sqrt{1 + \cos x^2} dx$ | (D) None

Q270 Find the length of the segment of the line $y = x$ between the points (a, a) and (b, b).

(A) $\sqrt{2}(b - a)$ | (C) $\sqrt{2}(b + a)$ | (E) None

(B) $\sqrt{2(b - a)}$ | (D) $b - a$

Word of the Week: "Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country." (*David Hilbert*)