Instructions: Keep all devices capable of communication turned off and out of sight.

Q1 Let $c^2z^2 = a^2x^2 + b^2y^2$. Then find the value of $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2}\frac{\partial^2 z}{\partial y^2}$

Q2 If $f(x,y) = \sqrt{x^2 - y^2}\sin^{-1}(\frac{y}{x})$ then find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

Q3 If $u = \ln(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

Q4 Find an equation of the set of all points equidistant from the points $A = (1,5,3)$ and $b = (6,2,-2)$. Describe the set.

Q5 Find a vector that has the same direction as $(-2,4,2)$ but has length 6.

Q6 Find parametric equations for the line through $A = (5,1,0)$ that is perpendicular to the plane $2x - y + z = 1$

Q7 Find and sketch the domain of

$$f(x,y) = \sqrt{x^2+y^2-1} + \ln(4-x^2-y^2)$$

Q8 Find equation of the tangent plane to the surface $z = e^{x^2-y^2}$ at the point $(1,-1,1)$.

Q9 Find z_x if $xyz = \cos(x+y+z)$

Q10 Suppose that $\nabla f = \langle 2xy+3x^2y^2, x^2+2x^3y\rangle$ and $f(1,2) = 3$. What is the tangent plane to the surface $z = f(x,y)$ at the point $(1,2,3)$?

Q11 Find Maclaurin series of the function: $f(x) = \frac{\ln(1+x^2)}{x}$.

Q12 Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{n^3x^n}{3^n}$

Q13 Find a parametric equation of the line through the points $(1,2,-8),(9,7,1)$

Q14 Let $f(x,y) = x^2 - xy + y$. Find the equation of the tangent plane to the graph at the point $(-2,1,7)$.

Q15 Find the gradient of the function $f(x,y) = \sqrt{x^2+y^2+z^2}$,

Q16 Find the equation of the line through $(1,0,6)$ an orthogonal to the plane $x + 3y + z = 5$.

Q17 Find the equation of the tangent plane to the graph of $f(x,y) = x^2 - xy + y$ at the point $(-2,1,7)$.

Q18 Find the equation of the tangent plane to the graph of $f(x,y) = x^2/(y+1)$ at the point $(6,-3,-18)$.

Q19 Find the gradient of the function $r = \sqrt{x^2+y^2+z^2}$,

Q20 Find the gradient of the function $f(x,y,z) = \sin r = \sin\sqrt{x^2+y^2+z^2}$.

Q21 Find the equation of the plane that goes through the points $p_1 = (1,2,3)$, $p_2 = (1,3,2)$, $p_3 = (-1,3,10)$

Q22 Find a power series representation for the function $f(x) = \frac{x}{x-3}$ and determine the interval of convergence.

Q23 Find a power series representation for the function $f(x) = \ln(1+x)$ and determine the radius of convergence.

Q24 Let $f(x,y,z) = x^2\ln(x-y+z)$

1. Evaluate $f(3,6,4)$
2. Find the domain of f
3. Find the range of f

Q25 Find the limit, $\lim_{(x,y)\to(0,0)}\frac{2x^2y}{x^4+y^2}$ if it exists, or show that the limit does not exist.

Q26 Find the limit, $\lim_{(x,y,z)\to(2,3,0)}[xe^z + \ln(2x-y)]$ if it exists, or show that the limit does not exist.

Q27 Find the partial derivatives of the function $f(x,y) = x^y$

Q28 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

1. $z = f(x)g(y)$
2. $z = f(xy)$
3. $z = f\left(\frac{x}{y}\right)$

Q29 Find the domain and range of the function $f(x,y)=\sqrt{x-y}$.

Q30 Find and sketch the domain of the function $f(x,y) = \sqrt{4-2x^2-y^2}$.

Q30 Determine the largest set on which the function is continuous.

$$f(x,y)=\begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 1 & \text{if } (x,y)=(0,0) \end{cases}$$

Q31 Find an equation of the tangent plane to the given surface $z = \sin(x+y)$ at the point $(1,-1,0)$

Q32 Find the directional derivative of the function $g(x,y,z)=z^3-x^2y$ at the given point $(1,6,2)$ in the direction of the vector $v=3i+4j+12k$.

Q33 Find the limit, $\lim\limits_{(x,y)\to(1,1)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ if it exists, or show that the limit does not exist.

Q34 Find the limit, $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(2x^2+3y^2)}{2x^2+3y^2}$ if it exists, or show that the limit does not exist.

Q35 Find the limit, $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2-3y^2}{2x^2+3y^2}$ if it exists, or show that the limit does not exist.

Q36 Let $f(x,y)=x\sin y$.

a. Compute all first and second order partial derivatives of f.

b. Find and classify the critical points of f.

c. Find the absolute maximum and minimum values of f in the rectangular region $-1\le x\le 1$, $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$.

Q37 What do the level curves of the function $f(x,y)=3x^2+4y^2$ look like? Are they circles, ellipses, diamonds, parabolas, or waves?

Q38 Could there be a function $f(x,y)$ with $\nabla f=\langle 2x^2+3y, 3+\sin x\rangle$? Why or why not?

Q39 Suppose that $\nabla f=\langle 2xy+3x^2y^2, x^2+2x^3y\rangle$ and $f(1,2)=3$. What is the tangent plane to the surface $z=f(x,y)$ at the point $(1,2,3)$?

Q40 Find an equation for the plane containing the points $(3,-1,2)$, $(2,0,5)$, and $(1,-2,4)$. Express your answer in standard form.

Q41 Let $f(x,y)=e^{3x+y}$, and suppose that $x=s^2+t^2$ and $y=2+t$. Find $\partial f/\partial s$ and $\partial f/\partial t$ by substitution and by means of the chain rule. Verify that the results are the same for the two methods.

Q42

Q43 Find the limit, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y}{x+y}$ if it exists, or show that the limit does not exist.

Q44 Find the limit, $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy^2}{x^2+8y^4}$ if it exists, or show that the limit does not exist.

Q45 Find the limit, $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y-2x^2-2y^2}{x^2+y^2}$

Q46 Find an equation of tangent plane to the graph of paraboloid $z=\frac{1}{2}x^2+\frac{1}{2}y^2+4$ at the point $(1,-1,5)$.

Q47 Find the level surface of $F(x,y,z)=x^2+y^2+z^2$ passing through $(1,1,1)$. Graph the gradient at the point.

Q48 Find the level surface of $F(x,y)=-x^2+y^2$ passing through $(2,3)$. Graph the gradient at the point.

Q49 Calculate the iterated integral

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}dydx$$

Q50 Evaluate the integral by reversing the order of integration.

$$\int_0^1\int_{x^2}^1 x^3\sin(y^3)dydx$$

Q51 Find the volume of the solid bounded by the cylinder $y^2+z^2=4$ and the planes $x=2y, x=0, z=0$ in the first octant.

Q52 Evaluate the double integral $\displaystyle\int\int_D xydA$, where D is the first-quadrant part of the disk with center $(0,0)$ and radius 1

Q53 Find the volume of the solid lying under the elliptic paraboloid $\dfrac{x^2}{4}+\dfrac{y^2}{9}+z=1$ and above the square $R=[-1,1]\times[-2,2]$.

Q54 Calculate the double integral

$$\int\int_R x\sin(x+y)dA, \qquad \text{where } R=\left[0,\frac{\pi}{6}\right]\times\left[0,\frac{\pi}{3}\right]$$

Q55 Evaluate $\int_0^1\int_0^{y^2} x^2y\,dx\,dy$ and sketch the region of integration in $\mathbb{R}^2$ indicated by the limits of integration.

Q56 Evaluate $\int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}\,dx\,dy.$

Q57 Let f be differentiable function and $z=e^{ax+by}f(ax-by)$. Then find $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}$

Q58 Divide 30 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

Q59

Evaluate

$$\iint_\Omega (x+y)^2\,dxdy$$

where Ω is the parallelogram bounded by the lines

$$x+y=0, x+y=1, 2x-y=0, 2x-y=3.$$

Q60

Evaluate

$$\iint_\Omega xy\,dxdy$$

where Ω is the first-quadrant region bounded by the curves

$$x^2+y^2=4, x^2+y^2=9, x^2-y^2=1, x^2-y^2=4.$$

Q61

Let

$$S=\left\{(x,y,z)\mid x^2+y^2+z^2\geq 1\right\},$$

Evaluate

$$\iiint_S \frac{1}{\left(x^2+y^2+z^2\right)^2}dxdydz$$

Q62 Evaluate $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dA$

Q63 Show that $\int_{-\infty}^{\infty} e^{-x^2}\,dx=\sqrt{\pi}$

Q63 Show that $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx=\sqrt{2}\sqrt{\pi}$

Q64 Find an equation of tangent plane to the graph of $f(x,y)=\arctan(xy^2)$ at the point $(1,1,\pi/4)$.

Q65 Compute $\iiint_E \sqrt{y^2+z^2}\,dV$ where E is the region bounded by the paraboloid $x=y^2+z^2$ and the plane $x=1$.

Q66 Compute $\iint_S f\,dA$ where S is the parallelogram spanned by $\langle 3,2\rangle$ and $\langle 1,5\rangle$ and $f(x,y)=xy$.

Q67

(i) Find a linear transformation $T:\mathbb{R}^2\to\mathbb{R}^2$ that maps unit circle $x^2+y^2=1$ into ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(ii) Use the above linear transformation and find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Q68 By changing the variables so that the region R is transformed into a rectangle, evaluate the integral

$$\iint_R \left(5(y-x^2)^4+6xy\right)\left(y+2x^2\right)\,dxdy,$$

where R is the region in the first quadrant bounded by the curves

$$y=x^2,\quad y=x^2+1,\quad xy=1,\quad xy=2.$$

Q69 Evaluate the integral $\int_0^2\int_{2y}^4 \exp(-x^2)\,dxdy.$

Q70 T is the triangle in the x-y plane with vertices $(0,0)$, $(0,3)$, $(1,3)$. Evaluate the integral $\iint_T 6\exp(-y^2)\,dxdy.$

Q71 Find the volume of the given solid. Below $z=1-x^2$ and over the region $0\leq x\leq 1$, $0\leq y\leq x$.

Q72 Evaluate $\iint_R e^{x^2+y^2}dA$ where $R=\{(x,y)|x^2+y^2\leq 4\}$.

Q73 Evaluate $\int_0^1\int_x^1 e^{y^2}dydx$.

Q74 Evaluate the iterated integral $\int_0^2\int_0^y 2xy\,dx\,dy$.

Q75 Evaluate the integral $\int_0^8\int_{x/2}^4 e^{y^2}\,dy\,dx$ by reversing the order of integration.

Q76 Solve the system $u=x+5y$ and $v=x+3y$ for x and y and compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

Q77 Evaluate the integral $\int\int_R 4(x+3y)(x+5y)dA$, where R is bounded by the lines $x+5y=-3$, $x+3y=0$, $x+5y=0$, $x+3y=4$.

Q78 Verify that Green's Theorem is true for the line integral

$$\oint_C -ydx+xdy$$

where C is the circle with center at the origin and radius 4.

Q79 Verify that Green's Theorem is true for the line integral

$$\oint_C x^2ydx+xy^2dy$$

where C is the triangle with vertices $(0,0),(1,0),(1,3)$. Sketch the triangle.

Q80 Evaluate

$$\oint_C (1+\tan x)\,dx+\left(x^2+e^y\right)dy$$

Where C is the positively oriented boundary of the region R enclosed by the curves $y=\sqrt{x}$, $x=1$, and $y=0$. Be sure to sketch C.

Q81 Evaluate

$$\oint_C x^2ydx-xy^2dy$$

where C is the circle $x^2+y^2=4$ with counterclockwise orientation.

Q82 Evaluate the line integral

$$\int_C (x^2-y^2)dx+2xydy$$

along the curve C whose parametric equations are

$$x=t^2;\qquad y=t^3;\qquad 0\leq t\leq\frac{3}{2}$$

Q83 Evaluate the line integral

$$\oint_C \left(x^3+2y\right)dx+\left(4x-3y^2\right)dy$$

where C is the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

Q84 Evaluate

$$\oint_C (1+\tan x)\,dx+\left(x^2+e^y\right)dy$$

Where C is the positively oriented boundary of the region R enclosed by the curves $y=\sqrt{x}$, $x=1$, and $y=0$. Be sure to sketch C.

Q85

Q86

Q87

Q88

Q89

Q90 ...