Instructions: Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 15 min. **Multiple Choice Questions (65 points)**

Q1 Evaluate $\int_0^2 \int_1^x x^2 y \, dy \, dx$

(A) 28/15 | (C) 1/24 | (E) None
(B) 3 | (D) 1/13

Q2 Evaluate $\int_0^e \int_0^e \int_0^e \frac{1}{xyz} \, dx \, dy \, dz$

(A) π | (C) e | (E) None
(B) $\ln 2 + 1$ | (D) $\sqrt{2}e$

Q3 Evaluate the line integral

$$\oint_C (x^2 + y^2)dx + 2xydy,$$

where C is the square bounded by the lines $x = 0, x = 2, y = 0, y = 2$.

(A) -1 | (C) 1/2 | (E) None
(B) 0 | (D) 13

Q4 Find the directional derivative of the function $g(x, y) = (x + 3y)^2$ at the given point $(1, -1)$ in the direction of the vector $\overrightarrow{v} = \frac{1}{\sqrt{2}}(1, -1)$.

(A) $\sqrt{2} + 2$ | (C) $4\sqrt{2}$ | (E) None
(B) $\sqrt{3} - 1$ | (D) 5

Q5 Let $z = 4 + x^3 + y^3 - 3xy$. Which of the following statements are true?

1. $(1, 1)$ is a local maximum,

2. $(0, 0)$ is a saddle point

3. $(2, 4)$ is a local minimum.

(A) Only 1 | (C) Only 3 | (E) None
(B) Only 2 | (D) 1 and 3

Q6 Find $\lim_{t \to 0} \left[\frac{\sin t}{t} \overrightarrow{i} + t \overrightarrow{j} + (t - 1)^4 \overrightarrow{k} \right]$

(A) $\overrightarrow{i}$ | (C) $\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$ | (E) None
(B) $\overrightarrow{i} + \overrightarrow{j}$ | (D) $\overrightarrow{i} - \overrightarrow{k}$

Q7 What is the length of the arc described $r(t) = (3t^2 \overrightarrow{i} + 2t^3 \overrightarrow{j} + \overrightarrow{k})$

(A) $4\sqrt{2}$ | (C) $4\sqrt{2} - 2$ | (E) None
(B) $4\sqrt{2} + 1$ | (D) $\sqrt{2} + 4$

Q8 Compute $\int_{-1}^1 \sqrt{1 - x^2} dx$

(A) $\frac{\pi}{2}$ | (C) π^2 | (E) None
(B) 2π | (D) $2\pi^2$

Q9 Evaluate the series $\sum_{n=1}^{\infty} 2^{4-3n}$

(A) 125/9 | (C) 63/7 | (E) None
(B) 161/5 | (D) 128/7

Q10 Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$

(A) 1 | (C) 3 | (E) None
(B) 1.5 | (D) 4

Q11 The value of $\lim_{(x,y,z) \to (2,3,0)} [xe^z + \ln(2x - y)]$

(A) 2 | (B) $2\sqrt{2}$ | (C) 3 | (D) DNE | (E) None

Q12 Compute $\int \frac{dx}{x^2 - x}$.

(A) $\ln|\frac{x-1}{x}| + C$

(B) $2\ln|x| + \frac{1}{3}\ln|x^2 - x| + C$

(C) $\frac{1}{2}\ln|x^2 + 1| + C$

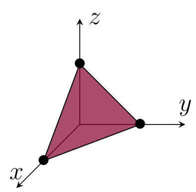
(D) None

Q13 Compute $\int_0^1 xe^x dx$.

(A) -1 | (B) 0 | (C) 1 | (D) 2 | (E) None

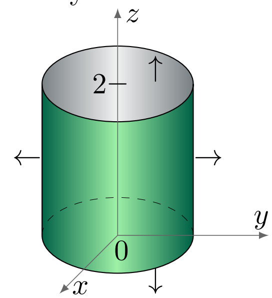
Classical Problems . Show all your work. No work=No credit!

Q1(15pts) Evaluate $\iint_S x dS$ where S is the triangle with vertices $(1,0,0), (0,1,0)$ and $(0,0,1)$.



Solution:

Q2(15pts) Consider the vector field $F = x\overrightarrow{i} + y\overrightarrow{j} + z\overrightarrow{k}$. Let S be the cylinder surface that lies between the planes $z = 0$ and $z = 2$.



Compute the surface integral: $\iint_R F \cdot dS$.

Solution:

Q3(10pts) Compute $\int\int_S F \cdot dS$ where S is the boundary of the cube: $E = [0,1]^3 = [0,1] \times [0,1] \times [0,1]$ and $F = \left(x + (y^2+1)^y, y + (z^2+1)^x, z + (x^2+1)^x\right)$.

Solution: