**Instructions:** Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 45 min. PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

Q1 Let $F(x) = \int_1^x \sqrt{2t^2 - 3t + 1}$. Find $F'(3)$.

(A) 10 | (C) $-1$ | (E) None
(B) 15 | (D) 0

Q2 What is the average of the function $h(t) = t^3 + 1$ on the interval $[1, 4]$ ?

(A) $-5$ | (C) 267/12 | (E) None
(B) 260/9 | (D) 211/13

Q3 Given that the area of the ellipse $4x^2 + y^2 = 4$ is $2\pi$ , evaluate the integral $\int_0^1 \sqrt{4 - 4x^2} dx$

(A) $-\pi$ | (C) $\pi$ | (E) None
(B) $\pi/2$ | (D) 1

Q4 Suppose that $f = g \circ h + h \circ g$, $g(0) = 1$, $h(0) = 2$, $g'(0) = 3$, $g'(2) = 5$, $h'(0) = 6$, $h'(1) = 7$ then $f'(0) =$

(A) 51 | (C) 61 | (E) None
(B) 55 | (D) 20

Q5 Evaluate the limit $\lim_{x \to 0} \dfrac{e^x - 1 - x}{2x^2}$

(A) $-2$ | (C) 0 | (E) None
(B) $-1$ | (D) 1/4

Q6 Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

(A) $-5$ | (C) 1 | (E) None
(B) $-1$ | (D) 3

Q7 Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

(A) $f(x)$ is not concave downward anywhere. | (B) $(0, 1)$ | (D) $(0, \infty$
(C) $(0, e)$ | (E) None

Q8 Suppose that a function $h(x)$ has derivative $h'(x) = x^2 + 4$. Find the $x$ value in the interval $[-1, 3]$ where $h(x)$ takes its minimum.

(A) 0 | (C) 2 | (E) None
(B) $-1$ | (D) 4

Q9 Evaluate the limit $\lim_{n \to \infty} \dfrac{2 + 4 + 6 \cdots + 2n}{n^2}$

(A) 2 | (C) 1.5 | (E) None
(B) 3 | (D) 1

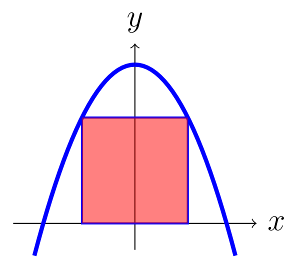
Q10 Evaluate the integral $\int_{-4}^{8} |x| dx$

(A) 24 | (C) 40 | (E) None
(B) 38 | (D) 48

Q11 Given the function $f(x) = x^2$, which value of $c$ satisfies the conclusion of the Mean Value Theorem on the interval $[-4, 5]$?

(A) 1/2 | (C) 3 | (E) None
(B) 2 | (D) 4

Q12 A rectangle in the $(x, y)$-plane has its base on the $x$-axis and its upper two corners on the curve $y = -x^2 + 3$. Find the largest possible area for such a rectangle.




(A) 4 | (C) 2.5 | (E) None
(B) 3 | (D) 2

Q13 Consider the function $f(x) = \dfrac{x^2+5x+6}{x^2-4}$. Which of the following statements describes the asymptotes of $f(x)$? (VA:Vertical asymptote, HA: Horizontal asymptote)

(A) VA: $x = \pm 2$, HA: $y = 1$.
(B) VA: $x = 2$, HA: $y = x$.
(C) VA: $x = \pm 2$, HA: $y = x$.
(D) VA: $x = 2$, HA: $y = 1$.
(E) None

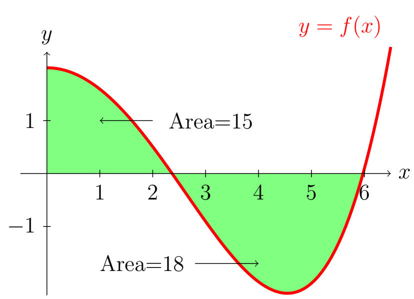
Q14 Find the linear approximation $L(x)$ to $f(x) = \cos x$ at $a = \frac{\pi}{2}$.

(A) $x + \frac{\pi}{3}$
(B) $x + \frac{\pi}{2}$
(C) $-x + \frac{\pi}{2}$
(D) $x + \frac{\pi}{3}$
(E) None

Q15 Suppose $f(t)$ is continuous on $[1, 7]$ with $f(1) = 1$ and $(7) = 49$. Find $\dfrac{d}{dt}\left[\int_1^7 f(t)dt\right]$

(A) 2
(B) 24
(C) 48
(D) 0
(E) None

Q16 Let $f(x)$ be an even function continuous on its domain $(-\infty, \infty)$. The figure below shows the areas of regions bounded by the graph of $f(x)$ and the $x-$ axis for $x$ in the interval $[0, 6]$. Find $\int_{-6}^{6} 2f(x)dx$.




(A) −12
(B) −10
(C) −6
(D) 10
(E) None

Q17 Find all possible functions with the given derivative $y' = \dfrac{9x}{\sqrt[5]{x}}$

(A) $9x^{9/5} + C$
(B) $5x^{9/5} + C$
(C) $5x^{9/4} + C$
(D) $6x^{9/5} + C$
(E) None

Q18 $f : (1.5, 9) \to \mathbb{R}$, $f(x^2 - 2) = 3x^2 - 2x + 2$. Then find $f'(6)$.

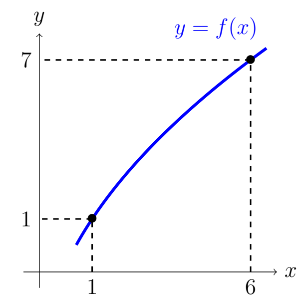
(A) 1
(B) 1.5
(C) 2
(D) 5/2
(E) None

Q19 Find the limit $\lim_{x\to 0}(1 + 3x)^{\frac{1}{x}}$

(A) $e^3$
(B) $3e$
(C) $e^{-2}$
(D) 2.71
(E) None

Q20 The graph of the function $f$ is shown right . Find the integral:

$$\int_1^6 f(x)dx + \int_1^7 f^{-1}(x)dx$$




(A) 38
(B) 39
(C) 41
(D) 42
(E) None

## Fill in the Blank Questions(10 pts)

Q1 The quantity $|y - x|$ is called the ________________ between ________ and ________ .

Q2 A function associates with each member of its ________________ precisely one member of its ________________

Q3 Suppose that $f$ is ________________ on $[a,\ b]$ and $f$ is ________________ on $(a,\ b)$ . Then there exists a constant ____ in $(a,\ b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Q4 The equation of the ________________ to the curve $y = f(x)$ at the point $x = a$ is given by $y - f(a) = f'(a)(x - a)$

Q5 $F(x)$ is an ________________ of $f(x)$ if $F'(x) = f(x)$ .

**True/False Questions(10 pts)** . No justifications are needed.

Q1 If $f(x)$ is continuous on $[a, b]$ and if $f(b) = f(a)$ then $f(x)$ must have a zero in $[a, b]$.

T F

Q2 If $f$ is constant 1 then $\int_a^b f(x)dx$ is the length of the interval $[a, b]$.

T F

Q3 If $f'(x) = g'(x)$, then $f(x) = g(x)$.

T F

Q4 The fundamental theorem of calculus implies that $\int_0^3 f''(x)dx = f'(3) - f(0)$.

T F

Q5 If $f(x)$ is smaller than $g(x)$ for all $x$, then $\int_0^1 f(x) - g(x)dx$ is negative.

T F

Q6 If $f$ is discontinuous at 0, then $\int_{-1}^1 f(x)dx$ is infinite.

T F

Q7 If $x(t) + y(t) = 10$ is constant and $x'(t) = 3$ then $y'(t) = -3$

T F

Q8 If a differentiable function $f$ has a critical point at 1, then the function $F(x) = \int_0^x f(t)dt$ has an inflection point at 1.

T F

Q9 The acceleration is the anti-derivative of the velocity.

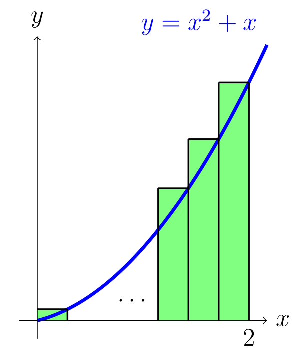
T F

Q10 If $f$ is concave up on $[0, 1]$ and concave down on $[1, 2]$ then 1 is an inflection points.

T F

**Bonus Question(15 pts)** .

Compute the definite integral $\int_0^2 (x + x^2)dx$ as the limit of a **right Riemann sum**.




(a) $\Delta x =$

(b) Right endpoint of the k-th subinterval, $x_k$ :

$=$

(c) Height of the k-th rectangle, $f(x_k)$

$=$

(d) Find the area of the k-th rectangle, $A_k = f(x_k)\Delta x$

$=$

(e) Sum of the areas of the n rectangles, $\sum_{k=1}^n A_k$

$=$

(f) Find the limit, $\lim_{n\to\infty} \sum_{k=1}^n A_k$

$=$