Instructions: Keep all devices capable of communication turned off and out of sight.The exam lasts for 1 hour and 15 min.

Q1 Find the sum of the series: $1+5+\frac{1}{2}+\frac{5}{3}+\frac{1}{4}+\frac{5}{9}+\frac{1}{8}+\frac{5}{27}+\cdots$

(A) diverges
(B) 19/2
(C) 15
(D) 43/2
(E) None

Q2 If $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$, then $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$ is equal to

(A) $\pi^2/12$
(B) $\pi^2/7$
(C) $\pi^2/8$
(D) $\pi^2/36$
(E) None

Q3 Consider the series $\sum_{n=1}^{\infty}\frac{1}{n3^n}$. Using the comparison test with the series leads to the following result. There is only one correct answer.

(A) The series converges
(B) The test is inconclusive
(C) The test diverges
(D) The test is not applicable
(E) None

Q4 The length of the curve determined by $x=2t^3$ and $y=t^3$ from $t=0$ to $t=1$ is

(A) 5/7
(B) $\sqrt{5}/2$
(C) $\sqrt{5}$
(D) 3
(E) None

Q5 Which of the following integrals represents the arclength of $y=\cos(x^2)$ over $[0,2]$?

(A) $2\pi\int_0^2\sqrt{1+\sin x^2}dx$
(B) $\int_0^2\sqrt{1+4x^2\cos x^2}dx$
(C) $2\pi\int_0^2\sqrt{1+36x^2\cos^2(x^2)}dx$
(D) $\int_0^2\sqrt{1+4x^2\sin^2(x^2)}dx$
(E) None

Q6 The length of the curve determined by : $x=\cos^3 t, y=\sin^3 t$ from $t=0$ to $t=1$ is

(A) 12
(B) 6
(C) $\sqrt{5}$
(D) 3
(E) None

Q7 Compute $\int\frac{dx}{x^3+x}$.

(A) $\ln|x|-\frac{1}{2}\ln|x^2+1|+C$
(B) $2\ln|x|+\frac{1}{3}\ln|x^2+1|+C$
(C) $\frac{1}{2}\ln|x^2+1|+C$
(D) None

Q8 Compute $\int_1^3\frac{x^3}{x+1}dx$.

(A) $\frac{21}{3}-\ln 2$
(B) $\frac{20}{3}-\ln 3$
(C) $\frac{22}{3}-\ln 2$
(D) $\frac{20}{5}-\ln 3$
(E) None

Q9 Compute $\int\frac{x^2}{x^2+1}dx$.

(A) $x+\arctan x+c$
(B) $x^2-\arctan x+c$
(C) $x-\arctan x+c$
(D) None

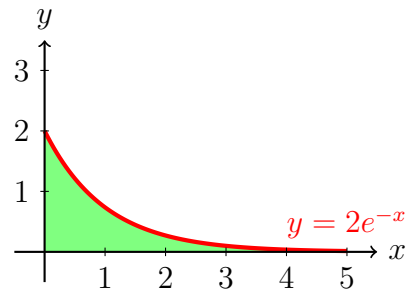
Q10 Compute $\int_{-a}^{a}\sqrt{a^2-x^2}dx$

(A) $\frac{\pi a^2}{2}$
(B) $a\pi$
(C) π^2
(D) $a\pi^2$
(E) None

Q11 The improper integral $\int_0^{\infty}xe^{-x}dx$

(A) diverges
(B) converges and equals 1
(C) converges and equals $\sqrt{2}$
(D) None

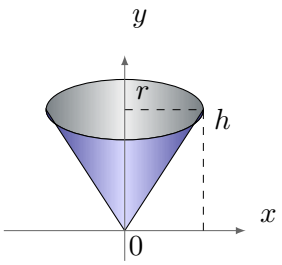
Q12 Find the volume of the solid generated when the region between the graphs of the functions $y = 2e^{-x}$, y-axis and the x-axis is revolved about the y-axis.



(A) π (C) 4π

(B) 2π (D) None

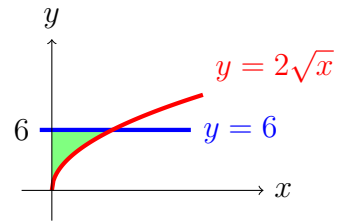
Q13 Find the surface area S of a right circular cone of height h and radius r.



(A) $2\pi\sqrt{h^2+r^2}$ (C) $\sqrt{h^2-r^2}$

(B) $\sqrt{h^2r^2}$ (D) None

Q14 The region bounded by $y = 2\sqrt{x}$, $y = 6$ and $x = 0$ is rotated about the y-axis to form a solid.Find the volume of the solid.



(A) $V = 386\pi/7$ (C) $V = 486\pi/5$

(B) $V = 236\pi/7$ (D) None

Q15 Let f be differentiable function. Which of the expression is the same as the integral $\int \frac{1}{x} f(x)dx$

(A) $f(x)\ln x - \int f'(x)\ln x dx$ (C) $f(x)\ln x - \int f(x)\ln x dx$

(B) $f(x)\ln x + \int f'(x)\ln x dx$ (D) None

Q16 Compute $\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$.

(A) $-\arcsin(\ln x) + c.$ (C) $\arcsin(\ln x) + c.$

(B) $\arcsin(\ln x) + c.$ (D) None

Q17 Compute $\int \frac{1}{64+x^2}\,dx$.

(A) $\frac{1}{6}\tan^{-1}\frac{x}{8} + c.$ (C) $\frac{1}{4}\tan^{-1}\frac{x}{4} + c.$ (E) None

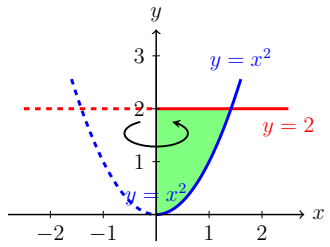
(B) $\frac{1}{18}\tan^{-1}\frac{x}{18} + c.$ (D) $\frac{1}{8}\tan^{-1}\frac{x}{8} + c.$

Q18 The integral $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}dx$ equals:

(A) $\pi/3$ (C) $3\pi/4$ (E) None

(B) $\pi/2$ (D) $\pi/4$

Q19 Find the surface area of the solid generated when the region between the graphs of the functions $y = x^2$,$y = 2$ and y-axis is revolved about the y-axis.



Solution: