Instructions: Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 15 min. **Multiple Choice Questions (65 points)**

Q1 Evaluate $\int_0^2 \int_1^x x^2 y\, dy\, dx$

(A) 28/15 | (C) 1/24 | (E) None
(B) 3 | (D) 1/13

Q2 Evaluate $\int_0^e \int_0^e \int_0^e \frac{1}{xyz}\, dx\, dy\, dz$

(A) π | (C) e | (E) None
(B) $\ln 2 + 1$ | (D) $\sqrt{2}e$

Q3 Evaluate the line integral

$$\oint_C (x^2 + y^2)dx + 2xydy,$$

where C is the square bounded by the lines $x = 0, x = 2, y = 0, y = 2$.

(A) -1 | (C) 1/2 | (E) None
(B) 0 | (D) 13

Q4 Find the directional derivative of the function $g(x, y) = (x + 3y)^2$ at the given point $(1, -1)$ in the direction of the vector $\vec{v} = \frac{1}{\sqrt{2}}(1, -1)$.

(A) $\sqrt{2} + 2$ | (C) $4\sqrt{2}$ | (E) None
(B) $\sqrt{3} - 1$ | (D) 5

Q5 Let $z = 4 + x^3 + y^3 - 3xy$. Which of the following statements are true?

1. $(1, 1)$ is a local maximum,

2. $(0, 0)$ is a saddle point

3. $(2, 4)$ is a local minimum.

(A) Only 1 | (C) Only 3 | (E) None
(B) Only 2 | (D) 1 and 3

Q6 Find $\lim_{t\to 0} \left[\frac{\sin t}{t}\vec{i} + t\vec{j} + (t-1)^4\vec{k}\right]$

(A) $\vec{i}$ | (C) $\vec{i} + \vec{j} + \vec{k}$ | (E) None
(B) $\vec{i} + \vec{j}$ | (D) $\vec{i} - \vec{k}$

Q7 What is the length of the arc described $r(t) = (3t^2\vec{i} + 2t^3\vec{j} + \vec{k})$

(A) $4\sqrt{2}$ | (C) $4\sqrt{2} - 2$ | (E) None
(B) $4\sqrt{2} + 1$ | (D) $\sqrt{2} + 4$

Q8 Compute $\int_{-1}^1 \sqrt{1 - x^2}dx$

(A) $\frac{\pi}{2}$ | (C) π^2 | (E) None
(B) 2π | (D) $2\pi^2$

Q9 Evaluate the series $\sum_{n=1}^{\infty} 2^{4-3n}$

(A) 125/9 | (C) 63/7 | (E) None
(B) 161/5 | (D) 128/7

Q10 Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$

(A) 1 | (C) 3 | (E) None
(B) 1.5 | (D) 4

Q11 The value of $\lim_{(x,y,z)\to(2,3,0)} [xe^z + \ln(2x - y)]$

(A) 2 | (C) 3 | (E) None
(B) $2\sqrt{2}$ | (D) DNE

Q12 Compute $\int \frac{dx}{x^2 - x}$.

(A) $\ln|\frac{x-1}{x}| + C$ | (C) $\frac{1}{2}\ln|x^2 + 1| + C$
(B) $2\ln|x| + \frac{1}{3}\ln|x^2 - x| + C$ | (D) None

Q13 Compute $\int_0^1 xe^x dx$.

(A) -1 | (C) 1 | (E) None
(B) 0 | (D) 2

True and False

Q1 The surface $x^2+y^2+z^2=2z$ is a sphere.

T F

Q2 The vector $(1,2,3)$ is perpendicular to the plane $2x+4y+6z=4$.

T F

Q3 $(0,0)$ is a local minimum of the function $f(x,y)=x^6+y^6$.

T F

Q4 If $|\overrightarrow{v}\times\overline{w}|=0$ then $\overrightarrow{v}=0$ or $\overrightarrow{w}=0$.

T F

Q5

T F

Q6 If the series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence a_n converges to 0.

T F

Q7 The $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is convergent.

T F

Q8 The $\int_1^{\infty}\frac{1}{x^2}dx$ is divergent.

T F

Q9 The intersection of the sphere $x^2+y^2+z^2=169$ with the plane $z=14$ is a circle.

T F

Q10 The vectors $\overrightarrow{v}=(2,1,5)$ and $\overrightarrow{w}=(2,1,-1)$ are perpendicular.

T F

Fill in the blanks

Q1 The ____________________ from the point $P=(x_1,y_1,z_1)$ to the ____________________ $ax+by+cz+d=0$ is given by

$$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}.$$

Q2 If $||\overrightarrow{v}||=1$ then $\overrightarrow{v}$ is called ____________________

Q3 A point (a,b) in the plane is called a ____________________ of a function $f(x,y)$ if $\nabla f(a,b)=(0,0)$.

Q4 A ____________________ of radius R centered at $P=(a,b,c)$ is the collection of points in space which have distance R from P.

Q5 If f_{xy} and f_{yx} are both ____________________ then $f_{xy}=f_{yx}$.

Classical Problems . Show all your work. No work=No credit!

Q1 Find the surface area of the part of the sphere $x^2+y^2+z^2=25$ that lies between the planes $z=3$ and $z=4$.

Solution:

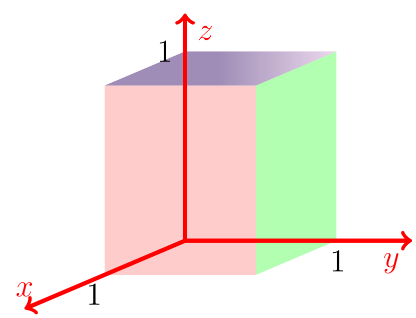
Q2 Let S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0$ and $z=1$. Let

$$\mathbf{F}(x,y,z)=4xz\mathbf{i}-y^2\mathbf{j}+yz\mathbf{k}$$

Find

$$\int\int_S F\cdot\mathbf{n}dS$$

where $\mathbf{n}$ is the unit outward normal to the surface S.



Solution: