Instructions: Keep all devices capable of communication turned off and out of sight.**Multiple Choice Questions (34 points)**

Q1 Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$ be finite sets. How many functions are there from A to B?

(A) 64 | (C) 81 | (E) None

(B) 32 | (D) 12

Q2 Find a constant c that makes $g(x)$ continuous on $(-\infty, \infty)$.

$$g(x) = \begin{cases} cx + 1 & \text{if } x \leq 2 \\ x + c & \text{if } x > 2 \end{cases}$$

(A) $c = 1$ | (C) $c = 2$ | (E) None

(B) $c = -\frac{1}{2}$ | (D) $c = 3$

Q3 What is $\lim_{h \to 0} \frac{(2+h)^5 - 2^5}{h}$

(A) 20 | (C) 60 | (E) None

(B) 40 | (D) 80

Q4 Suppose $f(x) = \frac{1}{x^2 - 5}$. What is the largest value of A such that $f(x)$ is defined on the interval $[-10, A)$?

(A) $-\sqrt{5}$ | (C) -2 | (E) None

(B) $2\sqrt{6}$ | (D) $\sqrt{5}$

Q5 If $f(x) = |x - 1|$ then find $\lim_{x \to 1} \frac{f(1+h) - f(1)}{h}$

(A) 4 | (C) 0 | (E) DNE

(B) 1 | (D) 2

Q6 The graph of a function f is reflected across the x-axis and then shifted up 2 units. Which of the following describes this transformation on f ?

(A) $-f(x)$ | (C) $-f(x) + 2$ | (E) None

(B) $-f(x - 2) + 2$ | (D) $-f(x - 2)$

Q7 $\lim_{x \to 1} \frac{\ln x}{x}$ is

(A) 0 | (C) e | (E) None

(B) 1 | (D) $-e$

Q8 $\lim_{x \to 0} \frac{\tan x}{x}$ is

(A) -1 | (C) π | (E) None

(B) $-\pi/2$ | (D) 1

Q9 $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}$ is

(A) -1 | (C) 1 | (E) None

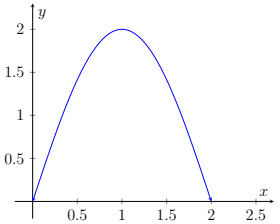
(B) 0 | (D) π

Q10 The Intermediate Value Theorem states that given a continuous function f defined on the closed interval $[a, b]$ for which 0 is between $f(a)$ and $f(b)$, there exists a point c between a and b such that

(A) $f(c) = 0$ | (C) $f(a) = f(b)$ | (E) None

(B) $c = a - b$ | (D) $f(0) = c$

Q11



The figure shows the graph od a sine function for one one complete period. Which of the following is an equation for the graph.?

(A) $f(x) = \sin x$ | (C) $f(x) = \sin(\pi x)$

(B) $f(x) = 2\sin(\frac{\pi}{2}x)$ | (D) $f(x) = 2\sin(\pi x)$

Q12 Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function.

(A) The function f is periodic. | (C) The function is strictly increasing function.

(B) The graph of f is symmetric with respect to the y-axis. | (D) The function is continuous.

Q13 Compute $\lim_{x\to 0} \dfrac{\tan x - x}{\sin x}$.

(A) 0 (C) 4/7 (E) None

(B) 1 (D) 5/9

Q14 Let $\dfrac{1}{5}x + \dfrac{5}{x} + 2 \le f(x) \le x^2 - 10x + 29$. Find the $\lim_{x\to 5} f(x)$

(A) -1 (C) 4 (E) 11

(B) 0 (D) 5

Q15 An object is moving along a straight line. It is position after $t \ge 0$ is given by $s(t) = t^2 + 5t$ meters. Find the average velocity of the object for the time interval $2 \le t \le 4$.

(A) -1 (C) 4 (E) 11

(B) 0 (D) 5

Q16 The slope of the line tangent to the graph of $y = x^3 - x^2 + 1$ at $x = 2$ is

(A) 8 (C) 11 (E) None

(B) 9 (D) 12

Q17 $\lim_{x\to\infty} \dfrac{5 + 7^x}{2 + 9^x}$

(A) ∞ (C) 5/2 (E) None

(B) 0 (D) 7/9

True/False questions(10 pts) . No justifications are needed.

Q1 If $f(x)$ is continuous on $[a, b]$ and if $f(b) = f(a)$ then $f(x)$ must have a zero in $[a, b]$.

T F

Q2 If $f'(x) = g'(x)$, then $f(x) = g(x)$.

T F

Q3 If $\lim_{x\to\infty} f(x)$ exists, then $f(x)$ must have a horizontal asymptote.

T F

Q4 Speed is the rate of change of acceleration over time.

T F

Q5 It is impossible for a function to have both a horizontal asymptote and a vertical asymptote.

T F

Q6 If $f(x)$ is a continuous function then f is differentiable.

T F

Q7 The derivative of $\tan(x)$ is $\sec(x)\tan(x)$.

T F

Q8 A circle can be the graph of a function.

T F

Q9 The equation $x^4 + x - 3 = 0$ has a solution in $[1, 2]$.

T F

Q10 If $f(x) = 7 + x + e^x$. Then $f^{-1}(8) = 0$.

T F

Fill in the Blank questions(20 pts) .

Q1 The quantity $|y - x|$ is called the _____ between _____ and _____ .

Q2 The function $f : x \to 3 - x$ _____ 2 onto 1.

Q3 A function associates with each member of its _____ precisely one member of its _____

Q4 A point (x, y) lies on the x-axis if and only if _____

Q5 A point (x, y) lies in the first quadrant if and only if both x and y are _____ .

Q6 Suppose that f is a _____ function on the interval $[a.b]$. Then, for every number w _____ $f(a)$ and $f(b)$, there is a c between a and b such that $f(c) = w$.

Q7 Let f be continuous on $[a,b]$ and _____ in (a,b). There is a c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} \,. \tag{2.1}$$

Q8 The _____ between the ***points*** $A(x_1,y_1)$ and $B(x_2,y_2)$ in the plane is

$$d(A,B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Q9 If $f'(a)$ exists then $\lim_{x\to a} f(x) =$ _____.

Q10 If f is a continuous function on a bounded closed interval $[a,b]$, then f has a _____ value M and minimum _____ m on the interval $[a,b]$.

Show all your work. No work=No credit (40 pts) .

Q1 Evaluate the following limit (without using L'Hopital's Rule):

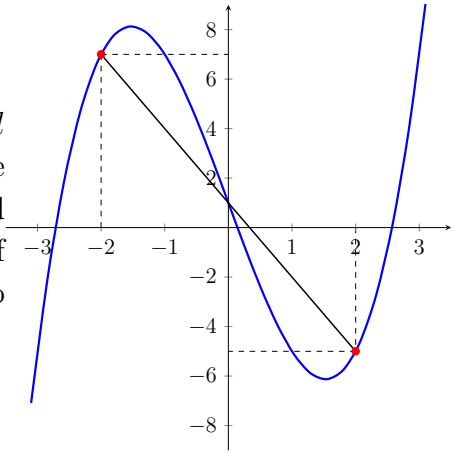
$$\frac{\tan(7x)}{2x}$$

Solution.

Q2 Without using derivatives, find $\lim_{x\to 0} \frac{1-\cos x}{x}$.

Solution.

Q3 Let $f(x) = x^3 - 7x + 1$ and let l be the secant line passing through the points $(2,(f(-2))$ and $(2,(f(2))$. Find all values of x such that the slope of the tangent line through x is equal to the slope of the secant line.



Solution.

Q4 Let $f(x) = \cos x$. Using the **definition of derivative**, show that

$$f'(0) = 0.$$

Solution.

Q5 Use the Intermediate-Value Theorem to show that the equation

$$x^3 - 2x^2 - 3x = -1$$

has at least two distinct solutions in the interval $[-2, 1]$.

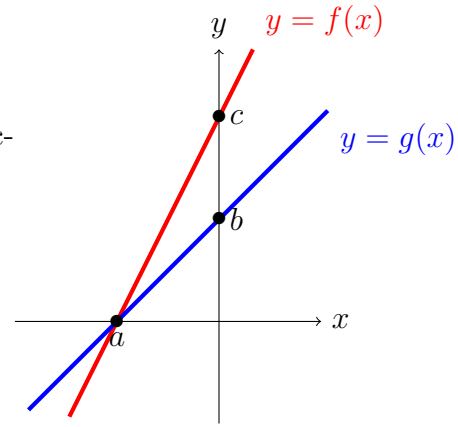
Solution.

Q6 Find all vertical, horizontal, and slant asymptotes for the following function: $\dfrac{2x^3 - 16}{x^2 - 7x + 6}$

Solution.

Q7 Suppose you have two linear function f and g shown right. Find

$$\lim_{x \to a} \frac{f(x)}{g(x)}.$$



Solution.

Q8 Suppose that x and y are functions of t which satisfy the relation

$$x^3y^2 + 2y = 8.$$

Suppose that at the point $(1, 2)$, the velocity of x is 3 in/sec. What is the velocity of y?

Solution.