Instructions: Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 45 min. PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

Q1 Let $F(x) = \int_1^x \sqrt{2t^2 - 3t + 1}$. Find $F'(3)$.

(A) 10
(B) 15
(C) -1
(D) 0
(E) None

Q2 What is the average of the function $h(t) = t^3 + 1$ on the interval $[1, 4]$?

(A) -5
(B) $260/9$
(C) $267/12$
(D) $211/13$
(E) None

Q3 Given that the area of the ellipse $4x^2 + y^2 = 4$ is 2π , evaluate the integral $\int_0^1 \sqrt{4 - 4x^2}dx$ (Hint: Think of the definite integral as an area.)

(A) $-\pi$
(B) $\pi/2$
(C) π
(D) 1
(E) None

Q4 Suppose that $f = g \circ h + h \circ g$, $g(0) = 1$, $h(0) = 2$, $g'(0) = 3$, $g'(2) = 5$, $h'(0) = 6$, $h'(1) = 7$ then $f'(0) =$

(A) 51
(B) 55
(C) 61
(D) 20
(E) None

Q5 Evaluate the limit $\lim_{x \to 0} \frac{e^x - 1 - x}{2x^2}$

(A) -2
(B) -1
(C) 0
(D) $1/4$
(E) None

Q6 Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

(A) -5
(B) -1
(C) 1
(D) 3
(E) None

Q7 Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

(A) $f(x)$ is not concave downward anywhere.
(B) $(0, 1)$
(C) $(0, e)$
(D) $(0, \infty$
(E) None

Q8 Suppose that a function $h(x)$ has derivative $h'(x) = x^2 + 4$. Find the x value in the interval $[-1, 3]$ where $h(x)$ takes its minimum.

(A) 0
(B) -1
(C) 2
(D) 4
(E) None

Q9 Evaluate the limit $\lim_{n \to \infty} \frac{2 + 4 + 6 \cdots + 2n}{n^2}$

(A) 2
(B) 3
(C) 1.5
(D) 1
(E) None

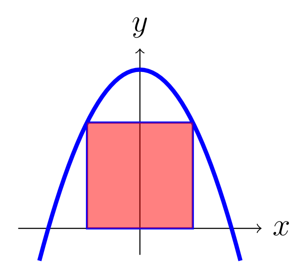
Q10 Evaluate the integral $\int_{-4}^{8} |x| dx$

(A) 24
(B) 38
(C) 40
(D) 48
(E) None

Q11 Given the function $f(x) = x^2$, which value of c satisfies the conclusion of the Mean Value Theorem on the interval $[-4, 5]$?

(A) $1/2$
(B) 2
(C) 3
(D) 4
(E) None

Q12 A rectangle in the (x, y)-plane has its base on the x-axis and its upper two corners on the curve $y = -x^2 + 3$. Find the largest possible area for such a rectangle.



(A) 4
(B) 3
(C) 2.5
(D) 2
(E) None

Q13 Consider the function $f(x) = \dfrac{x^2+5x+6}{x^2-4}$. Which of the following statements describes the asymptotes of $f(x)$? (VA:Vertical asymptote, HA: Horizontal asymptote)

(A) VA: $x = \pm 2$, HA: $y = 1$.

(B) VA: $x = 2$, HA: $y = x$.

(C) VA: $x = \pm 2$, HA: $y = x$.

(D) VA: $x = 2$, HA: $y = 1$.

(E) None

Q14 Find the linear approximation $L(x)$ to $f(x) = \cos x$ at $a = \frac{\pi}{2}$.

(A) $x + \frac{\pi}{3}$

(B) $x + \frac{\pi}{2}$

(C) $-x + \frac{\pi}{2}$

(D) $x + \frac{\pi}{3}$

(E) None

Q15 Suppose $f(t)$ is continuous on $[1, 7]$ with $f(1) = 1$ and $(7) = 49$. Find $\dfrac{d}{dt}\left[\int_1^7 f(t)dt\right]$

(A) 2

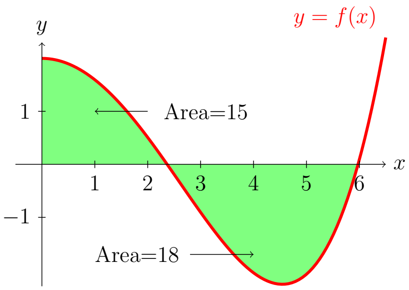
(B) 24

(C) 48

(D) 0

(E) None

Q16 Let $f(x)$ be an even function continuous on its domain $(-\infty, \infty)$. The figure below shows the areas of regions bounded by the graph of $f(x)$ and the $x-$ axis for x in the interval $[0, 6]$. Find $\int_{-6}^{6} 2f(x)dx$.



(A) -12

(B) -10

(C) -6

(D) 10

(E) None

Q17 Find all possible functions with the given derivative $y' = \dfrac{9x}{\sqrt[5]{x}}$

(A) $9x^{9/5} + C$

(B) $5x^{9/5} + C$

(C) $5x^{9/4} + C$

(D) $6x^{9/5} + C$

(E) None

Q18 $f : (1.5, 9) \to \mathbb{R}$, $f(x^2 - 2) = 3x^2 - 2x + 2$. Then find $f'(6)$.

(A) 1

(B) 1.5

(C) 2

(D) $5/2$

(E) None

Q19 Find the limit $\lim_{x\to 0}(1 + 3x)^{\frac{1}{x}}$

(A) e^3

(B) $3e$

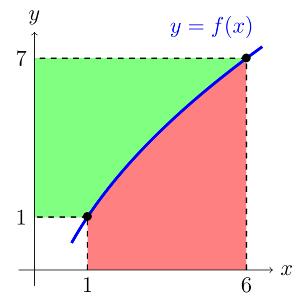
(C) e^{-2}

(D) 2.71

(E) None

Q20 The graph of the function f is shown right . Find the integral:

$$\int_1^6 f(x)dx + \int_1^7 f^{-1}(x)dx$$



(A) 38

(B) 39

(C) 41

(D) 42

(E) None

Fill in the Blank Questions(10 pts)

Q1 The quantity $|y - x|$ is called the ________________ between ________ and ________ .

Q2 A function associates with each member of its ________________ precisely one member of its ________________

Q3 Suppose that f is ________________ on $[a,\ b]$ and f is ________________ on $(a,\ b)$. Then there exists a constant ____ in $(a,\ b)$ such that $f\,'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Q4 The equation of the ________________ to the curve $y = f(x)$ at the point $x = a$ is given by $y - f(a) = f\,'(a)(x - a)$

Q5 $F(x)$ is an ________________ of $f(x)$ if $F\,'(x) = f(x)$.

True/False Questions(10 pts) . No justifications are needed.

Q1 If $f(x)$ is continuous on $[a,b]$ and if $f(b)=f(a)$ then $f(x)$ must have a zero in $[a,b]$.

T F

Q2 If f is constant 1 then $\int_a^b f(x)dx$ is the length of the interval $[a,b]$.

T F

Q3 If $f'(x)=g'(x)$, then $f(x)=g(x)$.

T F

Q4 The fundamental theorem of calculus implies that $\int_0^3 f''(x)dx = f'(3)-f(0)$.

T F

Q5 If $f(x)$ is smaller than $g(x)$ for all x, then $\int_0^1 f(x)-g(x)dx$ is negative.

T F

Q6 If f is discontinuous at 0, then $\int_{-1}^1 f(x)dx$ is infinite.

T F

Q7 If $x(t)+y(t)=10$ is constant and $x'(t)=3$ then $y'(t)=-3$

T F

Q8 If a differentiable function f has a critical point at 1, then the function $F(x)=\int_0^x f(t)dt$ has an inflection point at 1.

T F

Q9 The acceleration is the anti-derivative of the velocity.

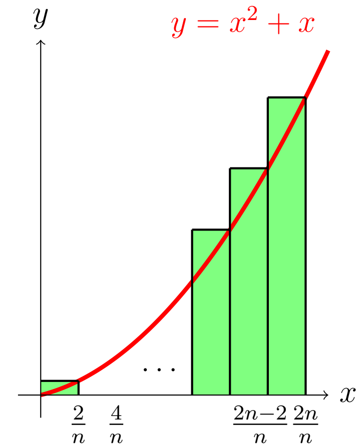
T F

Q10 If f is concave up on $[0,1]$ and concave down on $[1,2]$ then 1 is an inflection points.

T F

Show all your work. No work=No credit (40 pts) .

Q1 Compute the definite integral $\int_0^2 (x+x^2)dx$ as the limit of a **right Riemann sum**.



(a) $\Delta x =$

(b) Right endpoint of the k-th subinterval, $x_k =$

(c) Height of the k-th rectangle: $f(x_k) =$

(d) Find the area of the k-th rectangle: $A_k = f(x_k)\Delta x =$

(e) Sum of the areas of the n rectangles: $\sum_{k=1}^n A_k =$

(f) Find the limit: $\lim_{n\to\infty} \sum_{k=1}^n A_k =$

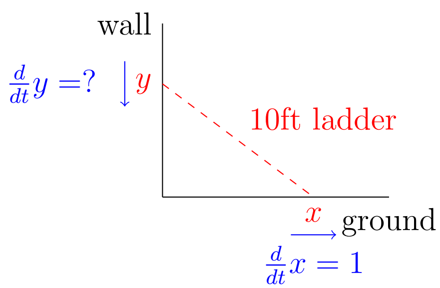
Q2 Let $f(x) = 2\tan x$.

- Find the linear approximation $L(x)$ of $f(x) = 2\tan x$ at $x = \pi$.
- Use linear approximation $L(x)$ to approximate $2\tan(3.3)$.

Q3 Find $\lim_{x\to\frac{\pi}{2}^-}(\tan x - \sec x)$

Q4 Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

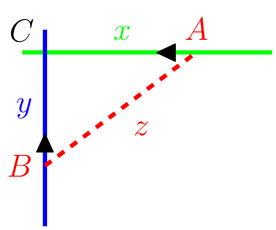
Q5 A ladder of length 10ft rests against a vertical wall. The bottom of the ladder slides away from the wall with 1ft/s How fast is the top sliding when the bottom is 6ft from the wall?



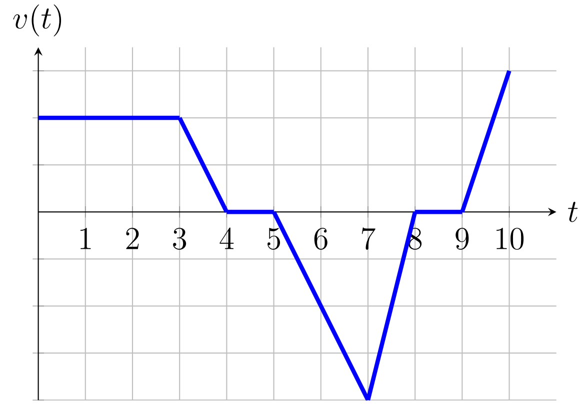
Q6 Two cars are headed for the same road intersection:

- car A is traveling west with 50mi/h
- car B is traveling north with 60mi/h

At what rate are the cars approaching
when A is 0.3mi and B is 0.4mi from the intersection?



Q7 The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in a closed interval $[0, 10]$.



Fill in the following blanks. Use interval notation where appropriate.

- The particle is moving forward during: ______________________
- The particle's speed is increasing during: ______________________
- The particle has positive acceleration during: ______________________
- The particle has zero acceleration during: ______________________
- The particle achieves its greatest speed at: ______________________
- The particle stands still for more than an instant during: ______________________