Instructions: Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 45 min. PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

Multiple Choice(85pts)

Q1 Find the sum of the series $\sum_{n=1}^{+\infty}\left(\frac{e}{\pi}\right)^n$

(A) $\frac{e}{\pi+1}$ (C) π (E) None

(B) $\frac{e}{\pi-e}$ (D) $\frac{\pi}{e-\pi}$

Q2 If $\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$, then $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}$ is equal to

(A) $\pi^4/96$ (C) $\pi^4/7$ (E) None

(B) $\pi^4/12$ (D) $\pi^2/36$

Q3 Consider the series $\sum_{n=1}^{\infty}\frac{1}{2^n+5n-2}$. Using the comparison test with the series leads to the following result. There is only one correct answer.

(A) The series converges (C) The test is not applicable

(B) The test diverges (D) None

Q4 Suppose f'' is continuous and f and f' have the values given below.

Evaluate $\int_1^3 xf''(x)dx$

	$x=1$	$x=2$	$x=3$
$f(x)$	2	5	8
$f'(x)$	3	1	4

(A) -12 (C) 3 (E) None

(B) 0 (D) 4

Q5 Find the power series for $\ln(1+x)$.

(A) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}$ (B) $\sum_{n=1}^{\infty}\frac{(-1)^{n+2}x^n}{n+1}$ (D) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$

(C) $\sum_{n=1}^{\infty}\frac{x^n}{n}$ (E) None

Q6 The length of the curve determined by : $x=\cos^3 t, y=\sin^3 t$ from $t=0$ to $t=\frac{\pi}{2}$ is

(A) 5/3 (C) 3/4 (E) None

(B) 6 (D) 3

Q7 The length of the curve determined by $x=3t$ and $y=2t^2$ from $t=0$ to $t=9$ is

(A) $\int_0^9\sqrt{9t^2+4t^4}dt$ (C) $\int_0^9\sqrt{9t^2+4t^4}dt$ (E) None

(B) $\int_0^9\sqrt{9+16t^2}dt$ (D) $\int_0^9\sqrt{9+16t^4}dt$

Q8 Compute $\int\frac{dx}{x^3-x}$.

(A) $\ln(\frac{\sqrt{x^2+x}}{|x|})+c$ $\frac{1}{3}\ln|x^2+1|+C$ (D) $\ln(\frac{\sqrt{x^2-1}}{|x|})+c$

(B) $2\ln|x|$ + (C) $\frac{1}{2}\ln|x^2+1|+C$ (E) None

Q9 Compute $\int_0^2\sqrt{4-x^2}dx$

(A) $\frac{\pi}{2}$ (C) $2\pi^2$ (E) None

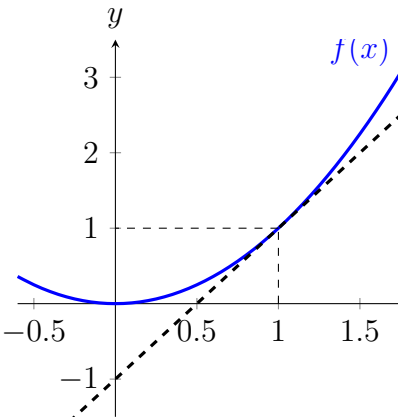
(B) π (D) 4π

Q10 Find the value of the definite integral $\int_0^1 xe^{-x}dx$

(A) $1-2e^{-1}$ (C) $-1+4e^{-1}$ (E) None

(B) $-1+2e^{-1}$ (D) $2e^{-1}$

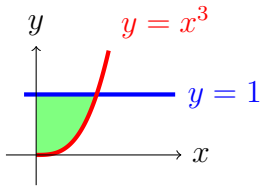
Q11 The graph of the function f is shown right . Find the integral $\int_0^1 f'(x)f''(x)dx$.



(A) -1 (C) 2 (E) None

(B) 1/2 (D) 5/2

Q12 The region bounded by $y = x^3$, $y = 1$ and $x = 0$ is rotated about the y-axis to form a solid.Find the volume of the solid.



(A) $V = 6\pi/7$
(B) $V = 4\pi/7$
(C) $V = 2\pi/7$
(D) $V = 3\pi/5$
(E) None

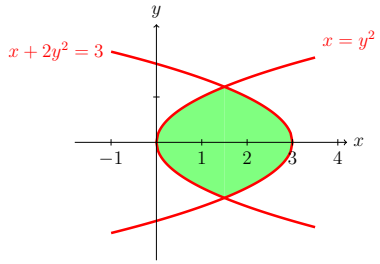
Q13 Compute $\int \frac{1}{3+x^2}\,dx$.

(A) $\frac{1}{6}\tan^{-1}\frac{x}{3} + c$.
(B) $\frac{1}{18}\tan^{-1}\frac{x}{18} + c$.
(C) $\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + c$.
(D) $\frac{1}{3}\tan^{-1}\frac{x}{9} + c$.
(E) None

Q14 The integral $\int_0^{\frac{\pi}{2}} \sin^2 x dx$ equals:

(A) $\pi/3$
(B) $\pi/2$
(C) $3\pi/4$
(D) $\pi/4$
(E) None

Q15 Find the area of the region between $x = y^2$ and $x + 2y^2 = 3$.



(A) 4
(B) 5
(C) 9
(D) 11
(E) None

Q16 Find $\lim_{n\to\infty} \cos\frac{\pi}{n}$.

(A) -1
(B) 0
(C) 1
(D) π
(E) None

Q17 Evaluate $\int \frac{\sin 4x}{\cos 2x} dx$.

(A) a
(B) b
(C) c
(D) d
(E) None

True and False(10pts)

Q1 If the series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence a_n converges to 0.

T F

Q2 The $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

T F

Q3 The $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$ is divergent.

T F

Q4 The $\sum_{n=1}^{\infty} n \sin\frac{1}{n}$ is divergent.

T F

Q5 The $\int_1^{\infty} \frac{1}{x^2} dx$ is divergent.

T F

Q6 If the graph of the function $f(x) = x^2$ is rotated around the interval $[0, 1]$ in the x axes we obtain a solid with volume $\int_0^1 \pi x^4 dx$.

T F

Q7 The integral $\int_{-1}^{1} \frac{1}{x^2} dx = -\frac{1}{x}|_{-1}^{1} = -2$ is defined and negative.

T F

Q8 Gabriels trumpet has finite volume but infinite surface area .

T F

Q9 The fundamental theorem of calculus implies that $\int_a^b f' dx = f(b) - f(a)$.

T F

Fill in the blanks(15pts)

Q1 The curves $\begin{cases} x = 3\sin t \\ y = 2 + 3\cos t \end{cases}$ $(0 \le t \le 2\pi)$ parametrize a circle of radius _____ centered _____ .

Q2 If the series $\sum_{k=1}^{\infty} a_k$ _____ , then $\lim_{k\to\infty} = 0$.

Q3 If $\lim_{k\to\infty} \neq 0$ then the series $\sum_{k=1}^{\infty} a_k$ _____ , then $\lim_{k\to\infty} = 0$

Q4 Write an integral that represents the surface area when the curve $y = \tan x$, $0 \le x \le 4$ is revolved about the line $y = -2$.
S= _____

Q5 $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$ _____

Classical problems(25pts) . Show all your work. No work=No credit

Q1 Find $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$

Q2 Use a comparison test to determine whether or not the series $\sum_{n=1}^{\infty} \frac{1}{e^n + n^4 + 4}$ converges.

Q3 Use the ratio test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^\pi}{\pi^n}$ converges or diverges.

Q4 Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$, and its sum there.

Q5 Find the area of the surface when the graph of $f(x) = \sqrt{x}$ between $x = 1$ and $x = 4$ and is revolved about the x-axis.

Q6 Find the arc length of the curve $\begin{cases} x = 2t^3 \\ y = 3t^2 \end{cases}$ for $0 \le t \le \sqrt{3}$.

Q7 Find the arc length of the curve $y = 10x^{\frac{3}{2}}$ for $0 \le x \le 3$.

Q8 Find the volume of the solid of revolution formed by rotating the region bounded by the x-axis and the graph of $f(x) = \sqrt[5]{x}$, from $x = 0$ to $x = 1$, about the x-axis.

Q9 Find the volume of the solid of revolution formed by rotating the region bounded by the y-axis and the graph of $f(x) = \sqrt[7]{x}$, from $y = 0$ to $y = 1$, about the y-axis.

Q10 Find the Maclaurin series expansion of $f(x) = \ln(1 - x)$, and its interval of convergence.

Q11 Find the Maclaurin series expansion of $f(x) = \frac{x}{1-x^3}$, and its interval of convergence.

Q12 Compute the sum: $1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} + \cdots$ ..

Q13 Evaluate $\int_1^e \ln x dx$

Q14 Find $\int \frac{\sin x}{1 - \cos^2 x} dx$

Q15 Evaluate $\int_0^4 \sqrt{4x - x^2} dx$.