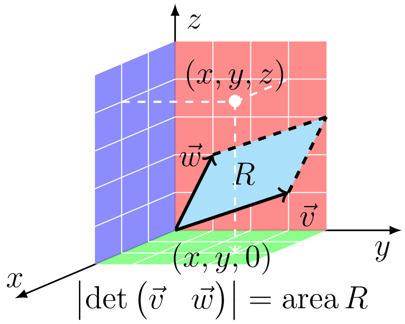

ENGINEERING MATHEMATICS-I
SAMPLE MIDTERM EXAM

April 2 th, 2017

Name: ________________________________

Multiple Choice(50pts)

Q1 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mappting, satisfying $T(1, 2) = (1, 0, 1)$ and $T(2, 5) = (0, 1, 1)$. Find $T(0, 1)$.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q2 If $u = (-2, 1, 1)$ and $v = (1, 0, 1)$, then $||\text{proj}_v u||$ is

(A) 0 | (B) 1/2 | (C) $\frac{1}{\sqrt{2}}$ | (D) 1 | (E) None

Q3 Parametric equation for the line passing through $(1, 1, -1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

(A) $x = 1 + 2t, y = 1 - t, z = -1 + 3t$

(B) $x =, y =, z =,$

(C) $x = 1 + t, y = 1 - t, z = -1 + t$

(D) $x = 1 + t, y = 1 - t, z = -1 + 3t$

(E) None

Q4 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mapping, satisfying $T(1, 2) = (1, 0, 1)$ and $T(2, 5) = (0, 1, 1)$. Find $T(0, 1)$.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q5 Under what condition can a vector (a, b, c) be written as a linear combination of $(1, 2, 0)$ and $(1, 1, 1)$.

(A) $a + b + c = 0$ | (B) $a + 2b + c = 0$ | (C) $2a - b - c = 0$ | (D) $a2 + b + 2c = 0$ | (E) None

Q6 The angle between $u = (0, 3, -3)$ and $v = (-2, 2, -1)$ is:

(A) $\pi/3$ | (B) $\pi/6$ | (C) $\pi/2$ | (D) $\pi/4$ | (E) None

Q7 Find an equation of the plane which passes through the point $(1, -7, 8)$ and which is perpendicular to the line whose parametric equations are: $x = 2 + 2t, y = 7 - 4t, z = -3 + t; t \in \mathbb{R}$

(A) $2x - 4y + z = -38$ | (B) $x - 4y + z = 8$ | (C) $x - 4y + z = -18$ | (D) $2x - y + z = 11880$ | (E) None

Q8 Suppose a linear system has augmented matrix $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & q & p \end{array}\right)$. Find all values of p and q such that this system has a unique solution.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q9 Which of the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation?.

(A) $T(x, y) = (x, y + 1)$ | (B) $T(x, y) = (x + 1, y)$ | (C) $T(x, y) = (y^2, xy)$ | (D) $T(x, y) = (x, 0)$

(A) $\begin{pmatrix} 4 \\ 58 \end{pmatrix}$ | (B) $\begin{pmatrix} 2 \\ 51 \end{pmatrix}$ | (C) $\begin{pmatrix} 11 \\ -7 \end{pmatrix}$ | (D) $\begin{pmatrix} 4 \\ 68 \end{pmatrix}$ | (E) None

Q11 Suppose that A is 3×4. Then the number of solutions to the system $A\mathbf{x} = \mathbf{0}$ is

(A) infinite | (B) one | (C) two | (D) zero | (E) None

True& False(20 pts)

Q1 A consistent linear system with 2 equations and 3 variables must have infinitely many solutions.

T F

Q2 Homogeneous systems are always consistent.

T F

Q2 There exists a linear transformation T that maps $(1, 0)$ to $(5, 3, 4)$ and maps $(3, 0)$ to $(1, 3, 2)$.

T F

Fill in the blanks(10pts

Q1 A subset $\{v_1, \ldots, v_d\}$ of $\mathbb{R}^n$ is ________________
if there are ________________ $a_1, \ldots, a_d \in \mathbb{R}$, not all zero, such that

$$a_1v_1 + a_2v_2 + \cdots + a_dv_d = 0.$$

Q2 Given two vectors $u, v \in \mathbb{R}^n$ which are ________________ then $||u + v||^2 = ||u||^2 + ||v||^2$.

Q3 A general $m \times n$ matrix A has m ________________ and n ________________ . The ________________ in the matrix are called the ________________ of A.

Q4 A system which has a ________________ is called ________________ . Otherwise it is ________________ inconsistent.

Q5 Any set of vectors containing the ________________ is linearly dependent

Q6 If T is a Linear transformation then $T(0) =$ ________________

Classical problems(20pts) . Show all your work. No work=No credit

Q1 Balance the following chemical reaction

$$CO_2 + H_2O \to C_6H_{12}O_6 + O_2.$$

Q2 Find the values of a and b such that the following system :

$$\begin{array}{ccccccc} x & + & y & & & = & 2 \\ x & + & 2y & & & = & 1 \\ 3x & + & 5y & + & a & = & b \end{array}$$

(i) is inconsistent;

(ii) has infinitely many solutions;

(iii) has a unique solution.

$\boxed{\text{Q3}}$ Identify the **elementary row** operation performed to obtain the new row-equivalent matrix.

$$\left[\begin{array}{rr|r} 3 & -1 & -4 \\ -4 & 7 & 9 \end{array}\right] \sim \left[\begin{array}{rr|r} 3 & -1 & -4 \\ 8 & 3 & -7 \end{array}\right]$$

$\boxed{\text{Q4}}$ Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x+y \\ x-2y \end{pmatrix}$$

Draw the image of the unit square under T, label all of its vertices.

$\boxed{\text{Q5}}$ Show that the vectors $u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ are linearly independent.