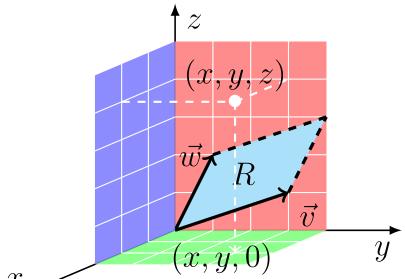

$\left|\det\left(\vec{v} \quad \vec{w}\right)\right| = \text{area}\, R$

ENGINEERING MATHEMATICS-I
SAMPLE FINAL EXAM

April 30, 2017

Name: ______________________________

Multiple Choice(50pts)

Q1 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mappting, satisfying $T(1, 2) = (1, 0, 1)$ and $T(2, 5) = (0, 1, 1)$. Find $T(0, 1)$.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q2 If $u = (-2, 1, 1)$ and $v = (1, 0, 1)$, then $||\text{proj}_v u||$ is

(A) 0 | (B) $1/2$ | (C) $\frac{1}{\sqrt{2}}$ | (D) 1 | (E) None

Q3 Parametric equation for the line passing through $(1, 1, -1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

(A) $x = 1 + 2t, y = 1 - t, z = -1 + 3t$

(B) $x =, y =, z =,$

(C) $x = 1 + t, y = 1 - t, z = -1 + t$

(D) $x = 1 + t, y = 1 - t, z = -1 + 3t$

(E) None

Q4 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mapping, satisfying $T(1, 2) = (1, 0, 1)$ and $T(2, 5) = (0, 1, 1)$. Find $T(0, 1)$.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q5 Under what condition can a vector (a, b, c) be written as a linear combination of $(1, 2, 0)$ and $(1, 1, 1)$.

(A) $a + b + c = 0$ | (B) $a + 2b + c = 0$ | (C) $2a - b - c = 0$ | (D) $a2 + b + 2c = 0$ | (E) None

Q6 The angle between $u = (0, 3, -3)$ and $v = (-2, 2, -1)$ is:

(A) $\pi/3$ | (B) $\pi/6$ | (C) $\pi/2$ | (D) $\pi/4$ | (E) None

Q7 Find an equation of the plane which passes through the point $(1, -7, 8)$ and which is perpendicular to the line whose parametric equations are: $x = 2 + 2t, y = 7 - 4t, z = -3 + t; t \in \mathbb{R}$

(A) $2x - 4y + z = -38$ | (B) $x - 4y + z = 8$ | (C) $x - 4y + z = -18$ | (D) $2x - y + z = 11880$ | (E) None

Q8 Suppose a linear system has augmented matrix $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & q & p \end{array}\right)$. Find all values of p and q such that this system has a unique solution.

(A) $(0, 2, 3)$ | (B) $(-1, 1, 2)$ | (C) $(0, 0, 1)$ | (D) $(1, 0, 0)$ | (E) None

Q9 Which of the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation?.

(A) $T(x, y) = (x, y+1)$ | (B) $T(x, y) = (x+1, y)$ | (C) $T(x, y) = (y^2, xy)$ | (D) $T(x, y) = (x, 0)$ (E) None

Q10 If $A = \begin{pmatrix} 3 & 2 \\ 5 & -4 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$, then $A\mathbf{x} =$

(A) $\begin{pmatrix} 4 \\ 58 \end{pmatrix}$ | (B) $\begin{pmatrix} 2 \\ 51 \end{pmatrix}$ | (C) $\begin{pmatrix} 11 \\ -7 \end{pmatrix}$ | (D) $\begin{pmatrix} 4 \\ 68 \end{pmatrix}$ | (E) None

Q11 Suppose that A is 3×4. Then the number of solutions to the system $A\mathbf{x} = \mathbf{0}$ is

(A) infinite | (B) one | (C) two | (D) zero | (E) None

Q12 Suppose that A is 2×2 with $\det(2BB^t) = 64$. Find $\det(3B^3B^t)$.

(A) 1 | (B) 8 | (C) 118 | (D) 80 | (E) None

Q13 The volume of a parallelepiped generated by $(1, 1, 0)$, $(1, 0, -1)$ and $(1, 1, 1)$ is:

(A) $\sqrt{3}$ | (B) 3 | (C) -2 | (D) 1 | (E) None

Q14 Find all p and q for which the linear system $\begin{cases} x - y = 3 \\ x + py = q \end{cases}$ has infinitely many solutions.

(A) $p = 2, q = 1$ | (B) $p = 1/2, q = -3$ | (C) $p = 2, q = -4$ | (D) $p = 1, q = 3$ | (E) None

Q15 Compute $(\det \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix})^{2017}$

(A) 1 | (B) 2 | (C) 118 | (D) 80 | (E) None

True& False(20 pts)

Q1 A consistent linear system with 2 equations and 3 variables must have infinitely many solutions.

T F

Q2 Homogeneous systems are always consistent.

T F

Q3 There exists a linear transformation T that maps $(1, 0)$ to $(5, 3, 4)$ and maps $(3, 0)$ to $(1, 3, 2)$.

T F

Q4 $\text{span}\{(4, 3, -1), (2, 1, 8)\} = \mathbb{R}^3$

T F

Q5 For all 2×2 matrices A and B, $(A + B)^t = A^t + B^t = B^t + A^t$.

T F

Q6 If A is an $m \times n$ matrix, then $\text{rank}A + \text{nullity}A = m$

T F

Q7 If A is an $n \times n$ matrix, then $\det(A + B) = \det A + \det B$

T F

Q8 If A is an $n \times n$ matrix, then $\det(\frac{A}{B}) = \frac{\det A}{\det B}$

T F

Q9 If A is an $n \times n$ matrix, then $\det(kA) = k\det A$

T F

Q10 If A is invertible matrix, then $\det(A^{-1}) = \det A$

T F

Q11 If A is not a invertible matrix, then $A\operatorname{adj}A \neq \det A$

T F

Q12 if $A = [a_{ij}]$ is an $n \times n$ triangular matrix, then $\det A = a_{11} + a_{22} + \cdots + a_{nn}$.

T F

Q13 C is said to be *similar* to A if there is an invertible matrix B such that $C = B^{-1}AB$. If C is similar to A then $\det A = \det C$

T F

Q14 If v_1, v_2, v_3 are linearly dependent, then $\operatorname{span}\{v_1, v_2\} \neq \operatorname{span}\{v_1, v_2, v_3\}$.

T F

Q15 Let $v_1, v_2, v_3 \in \mathbb{R}^3$. If $\operatorname{span}\{v_1, v_2, v_3\} \neq \mathbb{R}^3$, then v_1, v_2, v_3 are linearly dependent.

T F

Fill in the blanks(10pts

Q1 A subset $\{v_1, \ldots, v_d\}$ of $\mathbb{R}^n$ is ________________
if there are ________________ $a_1, \ldots, a_d \in \mathbb{R}$, not all zero, such that

$$a_1v_1 + a_2v_2 + \cdots + a_dv_d = 0.$$

Q2 Given two vectors $u, v \in \mathbb{R}^n$ which are ________________ then $||u + v||^2 = ||u||^2 + ||v||^2$.

Q3 A general $m \times n$ matrix A has m ________________ and n ________________ . The ________________ in the matrix are called the ________________ of A.

Q4 A system which has a ________________ is called ________________ . Otherwise it is ________________ inconsistent.

Q5 Any set of vectors containing the ________________ is linearly dependent

Q6 If T is a Linear transformation then $T(0) =$ ________________

Classical problems(20pts) . Show all your work. No work=No credit

Q1 Balance the following chemical reaction

$$CO_2 + H_2O \to C_6H_{12}O_6 + O_2.$$

Q2 Find the values of a and b such that the following system :

$$\begin{array}{rcrcrcl} x & + & y & & & = & 2 \\ x & + & 2y & & & = & 1 \\ 3x & + & 5y & + & a & = & b \end{array}$$

(i) is inconsistent;

(ii) has infinitely many solutions;

(iii) has a unique solution.

Q3 Identify the **elementary row** operation performed to obtain the new row-equivalent matrix.

$$\left[\begin{array}{rr|r} 3 & -1 & -4 \\ -4 & 7 & 9 \end{array}\right] \sim \left[\begin{array}{rr|r} 3 & -1 & -4 \\ 8 & 3 & -7 \end{array}\right]$$

Q4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x+y \\ x-2y \end{pmatrix}$$

Draw the image of the unit square under T, label all of its vertices.

Q5 Show that the vectors $u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ are linearly independent.

Q6 For any real numbers a, b, c, let $A = \begin{pmatrix} 1 & 2a & 4b \\ -2a & 1 & c \\ -4b & -c & 1 \end{pmatrix}$ and let $B = \begin{pmatrix} 0 & a^2 & b^2 \\ -a^2 & 0 & c^2 \\ -b^2 & -c^2 & 0 \end{pmatrix}$. Determine for which values of a, b, c, if any, A and B are invertible.

Q7 Suppose A is an 3×3 matrix such that $A^2 = 7A$.

1. What are the possible eigenvalues of A?
2. If A is invertible then find A.

Q8 For what value of a the matrix $A = \begin{pmatrix} 1 & a & 0 \\ 0 & 4 & 4 \\ a & 24 & 4 \end{pmatrix}$ is invertible.

Q9 Find the equation of the plane passing through the point $(1, 0, -1)$ and containing the line $x = 2+2t$, $y = 3+t$, $z = 4+3t$

Q10 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\begin{pmatrix} 1 \\ 2 \end{pmatrix}) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$, $T(\begin{pmatrix} -2 \\ 1 \end{pmatrix}) = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$. Find the matrix of T.

Q11 Let $A = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 6 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 6 \\ 1 & 0 \end{pmatrix}$. Find the matrix X such that $AXC + A = B^tC$.

Q12 Suppose A is a square matrix.

1. Define what is meant by an eigenvalue with corresponding eigenvector for the matrix A.

2. Explain or prove why the eigenvalues of A are found by solving the equation $|A - \lambda I| = 0$.

3. Suppose A is an invertible matrix. Is it possible for 0 to be an eigenvalue of A? Justify your answer.

Q13 Consider the matrix $A = \begin{bmatrix} 3 & -1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$.

1. Find the eigenvalues of A.

2. Find the corresponding eigenvectors.

Q14 Show that the function $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x, y, z) = (x - y, 2z)$ is a linear transformation.

Q15 Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0.

Q16 If $\det B = \left| \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \right| = 7$ then find the determinant of the If $\det C = \left| \begin{pmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{pmatrix} \right|$

Q17 Find the rank and nullity of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & 0 & 3 & 2 \\ 1 & 1 & 1 & -1 \\ -2 & -2 & 1 & 1 \end{pmatrix}$

Q18 Let $A = \begin{pmatrix} 1 & 2 & 1 \\ a & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}$. It is known that $\det A = 1$

1. Find a.

2. Find the determinant of $2A^t$.

3. Find A^{-1}.

Q19 Does the set of vectors $\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 11 \\ 25 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} \}$ spans $\mathbb{R}^3$