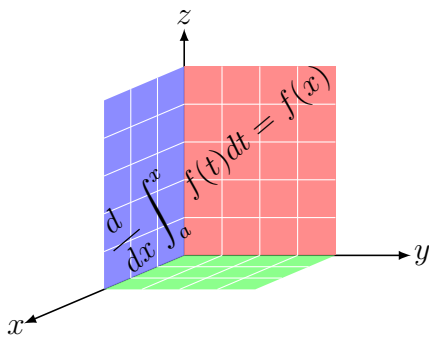


MATH121 MATHEMATICAL ANALYSIS-I  
SAMPLE MIDTERM EXAM

July 29, 2017



Name: \_\_\_\_\_

Be sure to show your work!

1 Evaluate each of the following limits.

1.  $\lim_{x \rightarrow 1} \frac{\sqrt{x+1} - \sqrt{2}}{x-1}$

2.  $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 4}$

3.  $\lim_{x \rightarrow \infty} \frac{3x}{\sqrt{7x^2 + x}}$

4.  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

5.  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$

6.  $\lim_{x \rightarrow \infty} \frac{\sin(\frac{3}{x})}{\frac{1}{x}}$

7.  $\lim_{x \rightarrow \frac{1}{2}} \sqrt{\frac{-4x^2 + 2x}{4x^2 - 8x + 3}}$

2 Given

$$f(x) = \begin{cases} 3x + 1 & \text{if } x < 2 \\ 4 - x^2 & \text{if } x \geq 2 \end{cases}$$

answer each of the following questions.

1.  $\lim_{x \rightarrow 2^+} f(x)$

2.  $\lim_{x \rightarrow 2^-} f(x)$

3.  $\lim_{x \rightarrow 2} f(x)$

4.  $f(2)$

3 Determine a value of  $k$  so that the function

$$f(x) = \begin{cases} 2kx + 9 & \text{if } x < 1 \\ k^2 - x^2 & \text{if } x \geq 1 \end{cases}$$

is continuous everywhere. Justify your steps.

4 Use the Intermediate Value Theorem to prove that the function  $f(x) = x^3 - x + 5$  has a zero.

5 Find the domain and sketch the graph of the function  $f(x) = \sqrt{9 - x^2}$

6 State the definition of continuity of a function  $f$  at  $x = c$ .

7 Determine the interval(s) on which  $f(x) = \sqrt{4 - x^2}$  is continuous.

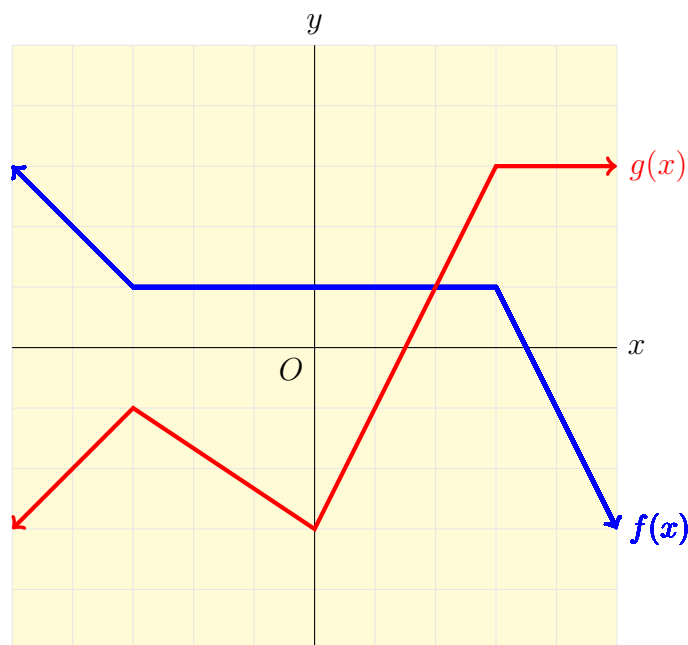
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The graphs of functions  $f$  and  $g$  are shown at right. Answer each of the following questions.

1.  $\lim_{x \rightarrow -3} (3f(x) - 5g(x))$

2.  $\lim_{x \rightarrow 0} (f(x)g(x))$

3.  $\lim_{x \rightarrow 0} \left( \frac{f(x)}{g(x)} \right)$



9 Determine all horizontal or slant asymptotes.

1.  $f(x) = \frac{1 - 2x}{\sqrt{3x^2 + 1}}$

2.  $f(x) = \frac{2x^3}{x^2 + 1}$

10 Let  $f(x) = \frac{3}{1 + 2x}$ .

1. Find the **slope of the secant line** to the curve  $y = f(x)$  passing through points where  $x = 1$  and  $x = 4$ .

2. Give the equation of this secant line.

11 Use Sandwich Theorem and limit laws to show that  $\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right) = 0$

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The graph of functions  $f$  is shown at right. Answer each of the following questions.

1. State the domain
2. Write the equations of any asymptotes

