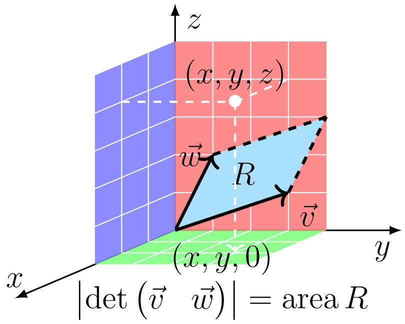

ENGINEERING MATHEMATICS-I
SAMPLE MIDTERM EXAM

April 30, 2017

Name: ________________________________

Multiple Choice(50pts)

Q1 Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mappting, satisfying $T(1,2) = (1,0,1)$ and $T(2,5) = (0,1,1)$. Find $T(0,1)$.

(A) $(0,2,3)$ | (B) $(-1,1,2)$ | (C) $(0,0,1)$ | (D) $(1,0,0)$ | (E) None

Q2 If $u = (-2,1,1)$ and $v = (1,0,1)$, then $||\text{proj}_v u||$ is

(A) 0 | (B) 1/2 | (C) $\frac{1}{\sqrt{2}}$ | (D) 1 | (E) None

Q3 Parametric equation for the line passing through $(1,1,-1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

(A) $x = 1+2t, y = 1-t, z = -1+3t$

(B) $x =, y =, z =,$

(C) $x = 1+t, y = 1-t, z = -1+t$

(D) $x = 1+t, y = 1-t, z = -1+3t$

(E) None

Q4 Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear mapping, satisfying $T(1,2) = (1,0,1)$ and $T(2,5) = (0,1,1)$. Find $T(0,1)$.

(A) $(0,2,3)$ | (B) $(-1,1,2)$ | (C) $(0,0,1)$ | (D) $(1,0,0)$ | (E) None

Q5 Under what condition can a vector (a,b,c) be written as a linear combination of $(1,2,0)$ and $(1,1,1)$.

(A) $a+b+c = 0$ | (B) $a+2b+c = 0$ | (C) $2a-b-c = 0$ | (D) $a2 + b + 2c = 0$ | (E) None

Q6 The angle between $u = (0,3,-3)$ and $v = (-2,2,-1)$ is:

(A) $\pi/3$ | (B) $\pi/6$ | (C) $\pi/2$ | (D) $\pi/4$ | (E) None

Q7 Find an equation of the plane which passes through the point $(1,-7,8)$ and which is perpendicular to the line whose parametric equations are: $x = 2+2t, y = 7-4t, z = -3+t; t \in \mathbb{R}$

(A) $2x - 4y + z = -38$ | (B) $x - 4y + z = 8$ | (C) $x - 4y + z = -18$ | (D) $2x - y + z = 11880$ | (E) None

Q8 Suppose a linear system has augmented matrix $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & q & p \end{array}\right)$. Find all values of p and q such that this system has a unique solution.

(A) $(0,2,3)$ | (B) $(-1,1,2)$ | (C) $(0,0,1)$ | (D) $(1,0,0)$ | (E) None

Q9 Which of the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation?.

(A) $T(x, y) = (x, y+1)$ | (B) $T(x, y) = (x+1, y)$ | (C) $T(x, y) = (y^2, xy)$ | (D) $T(x, y) = (x, 0)$ | (E) None

Q10 If $A = \begin{pmatrix} 3 & 2 \\ 5 & -4 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$, then $A\mathbf{x} =$

(A) $\begin{pmatrix} 4 \\ 58 \end{pmatrix}$ | (B) $\begin{pmatrix} 2 \\ 51 \end{pmatrix}$ | (C) $\begin{pmatrix} 11 \\ -7 \end{pmatrix}$ | (D) $\begin{pmatrix} 4 \\ 68 \end{pmatrix}$ | (E) None

Q11 Suppose that A is 3×4. Then the number of solutions to the system $A\mathbf{x} = \mathbf{0}$ is

(A) infinite | (B) one | (C) two | (D) zero | (E) None

Q12 Suppose that A is 2×2 with $\det(2BB^t) = 64$. Find $\det(3B^3B^t)$.

(A) 1 | (B) 8 | (C) 118 | (D) 80 | (E) None

Q13 The volume of a parallelepiped generated by $(1, 1, 0)$, $(1, 0, -1)$ and $(1, 1, 1)$ is:

(A) $\sqrt{3}$ | (B) 3 | (C) -2 | (D) 1 | (E) None

Q14 Find all p and q for which the linear system $\begin{cases} x - y = 3 \\ x + py = q \end{cases}$ has infinitely many solutions.

(A) $p = 2, q = 1$ | (B) $p = 1/2, q = -3$ | (C) $p = 2, q = -4$ | (D) $p = 1, q = 3$ | (E) None

Q15 Compute $(\det \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix})^{2017}$

(A) 1 | (B) 2 | (C) 118 | (D) 80 | (E) None

True& False(20 pts)

Q1 A consistent linear system with 2 equations and 3 variables must have infinitely many solutions.

T F

Q2 Homogeneous systems are always consistent.

T F

Q3 There exists a linear transformation T that maps $(1, 0)$ to $(5, 3, 4)$ and maps $(3, 0)$ to $(1, 3, 2)$.

T F

Q4 $\text{span}\{(4, 3, -1), (2, 1, 8)\} = \mathbb{R}^3$

T F

Q5 For all 2×2 matrices A and B, $(A + B)^t = A^t + B^t = B^t + A^t$.

T F

Q6 If A is an $m \times n$ matrix, then $\text{rank}A + \text{nullity}A = m$

T F

Q7 If A is an $n \times n$ matrix, then $\det(A + B) = \det A + \det B$

T F

$\boxed{\text{Q8}}$ If A is an $n \times n$ matrix, then $\det(\frac{A}{B}) = \frac{\det A}{\det B}$

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q9}}$ If A is an $n \times n$ matrix, then $\det(kA) = k\det A$

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q10}}$ If A is invertible matrix, then $\det(A^{-1}) = \det A$

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q11}}$ If A is not a invertible matrix, then $A\text{adj}A \neq \det A$

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q12}}$ if $A = [a_{ij}]$ is an $n \times n$ triangular matrix, then $\det A = a_{11} + a_{22} + \cdots + a_{nn}$.

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q13}}$ C is said to be *similar* to A if there is an invertible matrix B such that $C = B^{-1}AB$. If C is similar to A then $\det A = \det C$

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q14}}$ If v_1, v_2, v_3 are linearly dependent, then $\text{span}\{v_1, v_2\} \neq \text{span}\{v_1, v_2, v_3\}$.

$\boxed{\text{T}}$ $\boxed{\text{F}}$

$\boxed{\text{Q15}}$ Let $v_1, v_2, v_3 \in \mathbb{R}^3$. If $\text{span}\{v_1, v_2, v_3\} \neq \mathbb{R}^3$, then v_1, v_2, v_3 are linearly dependent.

$\boxed{\text{T}}$ $\boxed{\text{F}}$

Fill in the blanks(10pts

$\boxed{\text{Q1}}$ A subset $\{v_1, \dots, v_d\}$ of $\mathbb{R}^n$ is ____________________ if there are ____________________ $a_1, \dots, a_d \in \mathbb{R}$, not all zero, such that $a_1v_1 + a_2v_2 + \cdots + a_dv_d = 0$.

$\boxed{\text{Q2}}$ Given two vectors $u, v \in \mathbb{R}^n$ which are ____________________ then $||u+v||^2 = ||u||^2 + ||v||^2$.

$\boxed{\text{Q3}}$ A general $m \times n$ matrix A has m ____________________ and n ____________________ . The ____________________ in the matrix are called the ____________________ of A.

$\boxed{\text{Q4}}$ A system which has a ____________________ is called ____________________ . Otherwise it is ____________________ inconsistent.

$\boxed{\text{Q5}}$ Any set of vectors containing the ____________________ is linearly dependent

$\boxed{\text{Q6}}$ If T is a Linear transformation then $T(0) =$ ____________________

$\boxed{\text{Q7}}$ A linear equation is ____________________ if it is of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$.

$\boxed{\text{Q8}}$ A homogeneous equation (system) always has a ____________________ .

$\boxed{\text{Q9}}$ For an augmented matrix, the ____________________ are the first non-zero entries in each row of the matrix.

$\boxed{\text{Q10}}$ The ____________________ of a matrix is the number of pivots in its reduced row-echelon form.

$\boxed{\text{Q11}}$ ____________________ are those that corresponds to columns which are not pivot columns.

Q12 A linear equation is ________________ if it is of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$.

Q13 The matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

has rank 2. There are two________________ columns.

Classical problems(20pts) . Show all your work. No work=No credit

Q1 Balance the following chemical reaction

$$CO_2 + H_2O \to C_6H_{12}O_6 + O_2.$$

Q2 Find the values of a and b such that the following system :

$$\begin{array}{rcrcrcl} x & + & y & & & = & 2 \\ x & + & 2y & & & = & 1 \\ 3x & + & 5y & + & a & = & b \end{array}$$

(i) is inconsistent;

(ii) has infinitely many solutions;

(iii) has a unique solution.

Q3 Identify the **elementary row** operation performed to obtain the new row-equivalent matrix.

$$\left[\begin{array}{rr|r} 3 & -1 & -4 \\ -4 & 7 & 9 \end{array}\right] \sim \left[\begin{array}{rr|r} 3 & -1 & -4 \\ 8 & 3 & -7 \end{array}\right]$$

Q4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x+y \\ x-2y \end{pmatrix}$$

Draw the image of the unit square under T, label all of its vertices.

Q5 Show that the vectors $u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ are linearly independent.

Q6 For any real numbers a, b, c, let $A = \begin{pmatrix} 1 & 2a & 4b \\ -2a & 1 & c \\ -4b & -c & 1 \end{pmatrix}$ and let $B = \begin{pmatrix} 0 & a^2 & b^2 \\ -a^2 & 0 & c^2 \\ -b^2 & -c^2 & 0 \end{pmatrix}$. Determine for which values of a, b, c, if any, A and B are invertible.

Q7 Suppose A is an 3×3 matrix such that $A^2 = 7A$.

1. What are the possible eigenvalues of A?

2. If A is invertible then find A.

Q8 For what value of a the matrix $A = \begin{pmatrix} 1 & a & 0 \\ 0 & 4 & 4 \\ a & 24 & 4 \end{pmatrix}$ is invertible.

Q9 Find the equation of the plane passing through the point $(1, 0, -1)$ and containing the line $x = 2+2t$, $y = 3+t$, $z = 4+3t$

Q12 Suppose A is a square matrix.

1. Define what is meant by an eigenvalue with corresponding eigenvector for the matrix A.
2. Explain or prove why the eigenvalues of A are found by solving the equation $|A - \lambda I| = 0$.
3. Suppose A is an invertible matrix. Is it possible for 0 to be an eigenvalue of A? Justify your answer.

Q13 Consider the matrix $A = \begin{bmatrix} 3 & -1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$.

1. Find the eigenvalues of A.
2. Find the corresponding eigenvectors.

Q14 Show that the function $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x, y, z) = (x - y, 2z)$ is a linear transformation.

Q15 Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0.

Q16 If $\det B = \left| \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \right| = 7$ then find the determinant of the If $\det C = \left| \begin{pmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{pmatrix} \right|$

Q17 Find the rank and nullity of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & 0 & 3 & 2 \\ 1 & 1 & 1 & -1 \\ -2 & -2 & 1 & 1 \end{pmatrix}$

Q18 Let $A = \begin{pmatrix} 1 & 2 & 1 \\ a & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}$. It is known that $\det A = 1$

1. Find a.
2. Find the determinant of $2A^t$.
3. Find A^{-1}.

Q19 Does the set of vectors $\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 11 \\ 25 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix} \}$ spans $\mathbb{R}^3$

Q20 Find the rank of the matrix $A = \begin{pmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{pmatrix}$

Q21 Does the set $\{\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\}$ span $\mathbb{R}^3$

Q22 Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} -3 \\ 1 \\ 8 \end{pmatrix}, \vec{b} = \begin{pmatrix} h \\ -5 \\ -3 \end{pmatrix}$. For what value(s) of h is $\vec{b}$ in the plane generated by v_1 and v_2?

$\boxed{\text{Q23}}$ For what values of h is the set $T = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ of the vectors

$$\vec{w}_1 = (1, -1, 1) \qquad \vec{w}_2 = (0, 1, 2) \qquad \vec{w}_3 = (2, 3, h)$$

linearly dependent?

$\boxed{\text{Q24}}$ Find an equation involving $a, b,$ and c so that the following augmented matrix $\left(\begin{array}{ccc|c} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ -2 & 5 & -9 & c \end{array}\right)$ correspond to a consistent system.

$\boxed{\text{Q25}}$ Write an equation system that is equivalent to the vector equation:

$$x_1 \begin{pmatrix} 3 \\ -2 \end{pmatrix} + x_2 \begin{pmatrix} 7 \\ 3 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\boxed{\text{Q26}}$ Let $\mathbf{u} = (0, 4, 4)$ and $A = \begin{pmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{pmatrix}$ Is $\mathbf{u}$ in the plane spanned by the columns of A ? Why or why not?

$\boxed{\text{Q27}}$ Could a set of 4 vectors in $\mathbb{R}^5$ span all of $\mathbb{R}^5$? Explain.