Instructions: Keep all devices capable of communication turned off and out of sight. The exam lasts for 1 hour and 45 min. PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

Q1 Let $F(x) = \int_1^x \sqrt{2t^2 - 3t + 1}$. Find $F'(3)$.

(A) 10 (B) 15 (C) -1 (D) 0 (E) None

Q2 What is the average of the function $h(t) = t^3 + 1$ on the interval $[1, 4]$?

(A) -5 (B) 260/9 (C) 267/12 (D) 211/13 (E) None

Q3 Given that the area of the ellipse $4x^2 + y^2 = 4$ is 2π , evaluate the integral $\int_0^1 \sqrt{4 - 4x^2}dx$ (Hint: Think of the definite integral as an area.)

(A) $-\pi$ (B) $\pi/2$ (C) π (D) 1 (E) None

Q4 Suppose that $f = g \circ h + h \circ g$, $g(0) = 1$, $h(0) = 2$, $g'(0) = 3$, $g'(2) = 5$, $h'(0) = 6$, $h'(1) = 7$ then $f'(0) =$

(A) 51 (B) 55 (C) 61 (D) 20 (E) None

Q5 Evaluate the limit $\lim_{x\to 0} \frac{e^x - 1 - x}{2x^2}$

(A) -2 (B) -1 (C) 0 (D) 1/4 (E) None

Q6 Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

(A) -5 (B) -1 (C) 1 (D) 3 (E) None

Q7 Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

(A) $f(x)$ is not concave downward anywhere. (B) $(0, 1)$ (C) $(0, e)$ (D) $(0, \infty$ (E) None

Q8 Suppose that a function $h(x)$ has derivative $h'(x) = x^2 + 4$. Find the x value in the interval $[-1, 3]$ where $h(x)$ takes its minimum.

(A) 0 (B) -1 (C) 2 (D) 4 (E) None

Q9 Evaluate the limit $\lim_{n\to\infty} \frac{2 + 4 + 6 \cdots + 2n}{n^2}$

(A) 2 (B) 3 (C) 1.5 (D) 1 (E) None

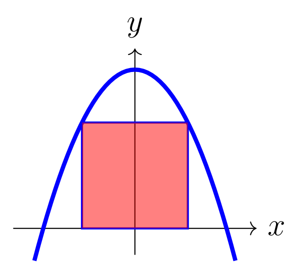
Q10 Evaluate the integral $\int_{-4}^{8} |x|dx$

(A) 24 (B) 38 (C) 40 (D) 48 (E) None

Q11 Given the function $f(x) = x^2$, which value of c satisfies the conclusion of the Mean Value Theorem on the interval $[-4, 5]$?

(A) 1/2 (B) 2 (C) 3 (D) 4 (E) None

Q12 A rectangle in the (x, y)-plane has its base on the x-axis and its upper two corners on the curve $y = -x^2 + 3$. Find the largest possible area for such a rectangle.



(A) 4 (B) 3 (C) 2.5 (D) 2 (E) None

Q13 Consider the function $f(x)=\dfrac{x^2+5x+6}{x^2-4}$. Which of the following statements describes the asymptotes of $f(x)$? (VA:Vertical asymptote, HA: Horizontal asymptote)

(A) VA: $x=\pm 2$, HA: $y=1$.
(B) VA: $x=2$, HA: $y=x$.
(C) VA: $x=\pm 2$, HA: $y=x$.
(D) VA: $x=2$, HA: $y=1$.
(E) None

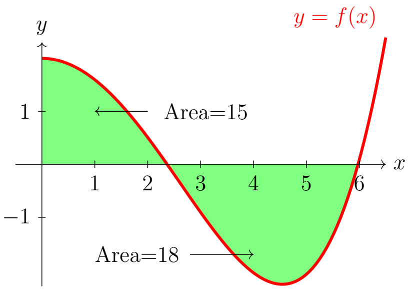
Q14 Find the linear approximation $L(x)$ to $f(x)=\cos x$ at $a=\frac{\pi}{2}$.

(A) $x+\frac{\pi}{3}$
(B) $x+\frac{\pi}{2}$
(C) $-x+\frac{\pi}{2}$
(D) $x+\frac{\pi}{3}$
(E) None

Q15 Suppose $f(t)$ is continuous on $[1,7]$ with $f(1)=1$ and $(7)=49$. Find $\dfrac{d}{dt}\left[\int_1^7 f(t)dt\right]$

(A) 2
(B) 24
(C) 48
(D) 0
(E) None

Q16 Let $f(x)$ be an even function continuous on its domain $(-\infty,\infty)$. The figure below shows the areas of regions bounded by the graph of $f(x)$ and the $x-$ axis for x in the interval $[0,6]$. Find $\int_{-6}^{6} 2f(x)dx$.



(A) -12
(B) -10
(C) -6
(D) 10
(E) None

Q17 Find all possible functions with the given derivative $y'=\dfrac{9x}{\sqrt[5]{x}}$

(A) $9x^{9/5}+C$
(B) $5x^{9/5}+C$
(C) $5x^{9/4}+C$
(D) $6x^{9/5}+C$
(E) None

Q18 $f:(1.5,9)\to\mathbb{R}$, $f(x^2-2)=3x^2-2x+2$. Then find $f'(6)$.

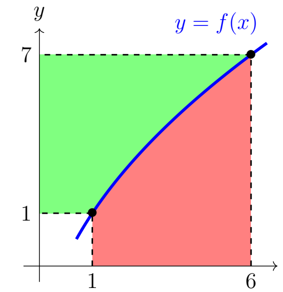
(A) 1
(B) 1.5
(C) 2
(D) 5/2
(E) None

Q19 Find the limit $\lim_{x\to 0}(1+3x)^{\frac{1}{x}}$

(A) e^3
(B) $3e$
(C) e^{-2}
(D) 2.71
(E) None

Q20 The graph of the function f is shown right . Find the integral:

$$\int_1^6 f(x)dx+\int_1^7 f^{-1}(x)dx$$



(A) 38
(B) 39
(C) 41
(D) 42
(E) None

Fill in the Blank Questions(10 pts)

Q1 The quantity $|y-x|$ is called the ________________ between ________ and ________ .

Q2 A function ________________ associates with each member of its ________________ precisely one member of its ________________

Q3 Suppose that f is ________________ on $[a,\ b]$ and f is ________________ on $(a,\ b)$. Then there exists a constant ____ in $(a,\ b)$ such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

Q4 The equation of the ________________ to the curve $y=f(x)$ at the point $x=a$ is given by $y-f(a)=f'(a)(x-a)$

Q5 $F(x)$ is an ________________ of $f(x)$ if $F'(x)=f(x)$.

True/False Questions(10 pts) . No justifications are needed.

Q1 If $f(x)$ is continuous on $[a,b]$ and if $f(b)=f(a)$ then $f(x)$ must have a zero in $[a,b]$.

T F

Q2 If f is constant 1 then $\int_a^b f(x)dx$ is the length of the interval $[a,b]$.

T F

Q3 If $f'(x)=g'(x)$, then $f(x)=g(x)$.

T F

Q4 The fundamental theorem of calculus implies that $\int_0^3 f''(x)dx = f'(3)-f(0)$.

T F

Q5 If $f(x)$ is smaller than $g(x)$ for all x, then $\int_0^1 f(x)-g(x)dx$ is negative.

T F

Q6 If f is discontinuous at 0, then $\int_{-1}^1 f(x)dx$ is infinite.

T F

Q7 If $x(t)+y(t)=10$ is constant and $x'(t)=3$ then $y'(t)=-3$

T F

Q8 If a differentiable function f has a critical point at 1, then the function $F(x)=\int_0^x f(t)dt$ has an inflection point at 1.

T F

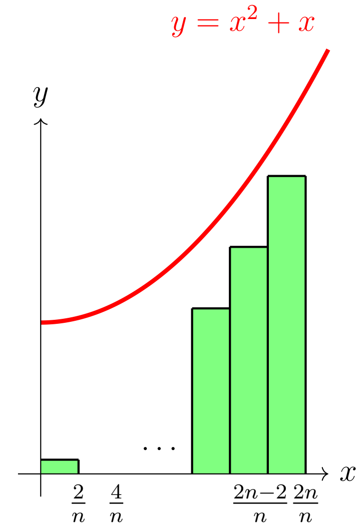
Q9 The acceleration is the anti-derivative of the velocity.

T F

Q10 If f is concave up on $[0,1]$ and concave down on $[1,2]$ then 1 is an inflection points.

T F

Show all your work. No work=No credit (40 pts) .



Q1 Compute the definite integral $\int_0^2 (x+x^2)dx$ as the limit of a **right Riemann sum**.

(a) $\Delta x =$

(b) Right endpoint of the k-th subinterval, $x_k =$

(c) Height of the k-th rectangle: $f(x_k)=$

(d) Find the area of the k-th rectangle: $A_k = f(x_k)\Delta x =$

(e) Sum of the areas of the n rectangles: $\sum_{k=1}^n A_k =$

(f) Find the limit: $\lim_{n\to\infty} \sum_{k=1}^{n} A_k =$

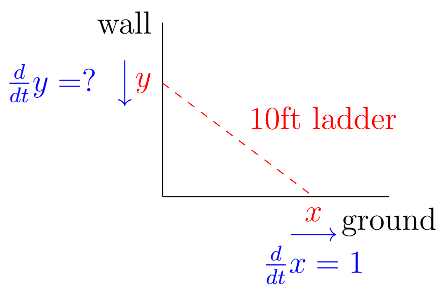
Q2 Let $f(x) = 2\tan x$.

- Find the linear approximation $L(x)$ of $f(x) = 2\tan x$ at $x = \pi$.
- Use linear approximation $L(x)$ to approximate $2\tan(3.3)$.

Q3 Find $\lim_{x\to\frac{\pi}{2}^-} (\tan x - \sec x)$

Q4 Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

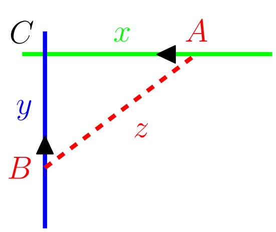
Q5 A ladder of length 10ft rests against a vertical wall. The bottom of the ladder slides away from the wall with 1ft/s How fast is the top sliding when the bottom is 6ft from the wall?



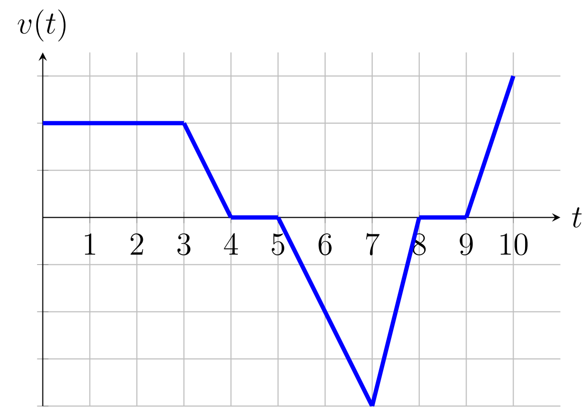
Q6 Two cars are headed for the same road intersection:

- car A is traveling west with 50mi/h
- car B is traveling north with 60mi/h

At what rate are the cars approaching
when A is 0.3mi and B is 0.4mi from the intersection?



Q7 The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in a closed interval $[0, 10]$.



Fill in the following blanks. Use interval notation where appropriate.

- The particle is moving forward during: ____________________
- The particle's speed is increasing during: ____________________
- The particle has positive acceleration during: ____________________
- The particle has zero acceleration during: ____________________
- The particle achieves its greatest speed at: ____________________
- The particle stands still for more than an instant during: ____________________

Q8 Evaluate the limit: $\lim_{n\to\infty} \sum_{i=1}^{n} e^{i\frac{4}{n}} \frac{4}{n}$.