Instructions: **Multiple Choice Questions (52 points)** Please Mark Your Answers With a Nice Circle ◯ NOT X!

Q1 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, satisfying $T(1,0) = (4,2)$ and $T(0,1) = (-2,-2)$. Find $T(2,1)$.

(A) $(2,3)$ (C) $(8,1)$ (E) None

(B) $(-1,-6)$ (D) $(6,2)$

Q2 If $u = (3,3,3)$ and $v = (4,2,6)$, then $\text{proj}_v u$ is

(A) $\frac{9}{7}(2,1,3)$ (C) $(1,1,1)$ (E) None

(B) $(2,1,3)$ (D) $(8,4,12)$

Q3 Given that $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is an eigenvector of $\begin{pmatrix} 3 & -2 \\ -4 & 1 \end{pmatrix}$ what is the corresponding eigenvalue?

(A) -1 (B) 0 (C) 2 (D) 9 (E) None

Q4 Find the eigenvalues of the matrix $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

(A) $(2,-5)$ (C) $(5,7)$ (E) None

(B) $(1,4)$ (D) $(-1,5)$

Q5 Suppose **u** and **v** are vectors in 3-space with $||u|| = 2$, $||v|| = 3$ and a 30° angle between them. Find the $\mathbf{u} \cdot \mathbf{v}$

(A) $\sqrt{6}$ (C) $3\sqrt{2}$ (E) None

(B) $2\sqrt{6}$ (D) $3\sqrt{3}$

Q6 Let A and B be 3×3 matrices such that $\det(A) = 10$ and $\det(B) = 12$. Find $\det(A^{-1}B^{-1}AB)$?

(A) -1 (B) 0 (C) 1 (D) 24 (E) None

Q7 Determine for which value(s) of a the matrix $\begin{pmatrix} 2 & a & a \\ a & a & a \\ 8 & 7 & a \end{pmatrix}$ is not invertible.

(A) -1 (B) 4 (C) 7 (D) 10 (E) None

Q8 Let A be 11×6 matrix such that $Ax = 0$ has only trivial solution $x = 0$. What is the rank of A.

(A) 5 (B) 6 (C) 11 (D) 17 (E) None

Q9 Let A be a 4×5 matrix such that $\text{rank}(A) = 4$. Then $\text{Nullity}(A)$ is

(A) 1 (B) 4 (C) 5 (D) 9 (E) None

Q10 Find the area of the parallelogram determined by the points $(0,0), (2,5), (6,1)$ and $(8,6)$.

(A) 18 (B) 36 (C) 32 (D) 28 (E) None

Q11 Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$. Then find $\text{rank}(A)$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) None

Q12 If $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 2$ then find the determinant $\det \begin{pmatrix} b+4c & e+4f & h+4i \\ -3c & -3f & -3i \\ 5a & 5d & 5g \end{pmatrix}$

(A) -40   (B) -30   (C) 10   (D) 24   (E) None

Q13 Define $T(x) = Ax$ where $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$. What is the image under T of $(4,1)$.

(A) $(3,1)$ (C) $(2,6)$ (E) None

(B) $(1,7)$ (D) $(7,9)$

True and False(10pts)

Q1 If $\{x_1, x_2, \cdots, x_n\}$ is linearly dependent and T is a linear transformation, then $\{T(x_1), T(x_2), \cdots, T(x_n)\}$ is also linearly dependent.

    T   F

Q2 Each invertible matrix A has the same eigenvectors as A^{-1}.

    T   F

Q3 If a 2 by 2 matrix A satisfies $A^2 = 0$ then $A = 0$.

    T   F

Q4 If A is a 4×5 matrix and B is a 5×3 matrix, then $\mathrm{rank}(A) \leq \mathrm{rank}(B)$.

    T   F

Q5 If A and B are 2×2 matrices then $(A+B)(A-B) = A^2 - B^2$.

    T   F

Classical Problems

Q1(16pts) Find the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

Solution:

Q2(22pts) Consider the map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2x \\ 3y \end{pmatrix}.$$

1. Draw the image of the disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$ under T.

2. Find the area of the image of the disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$ under T.   (Area of $T(D) =$?)

Solution:

$T : \mathbb{R}^2 \to \mathbb{R}^2$