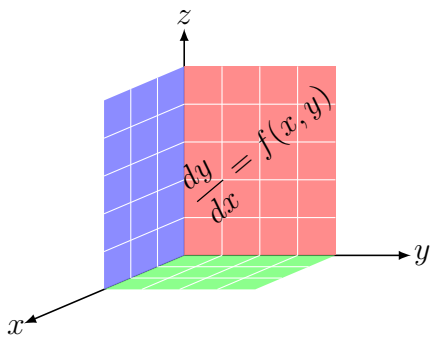


ENG227 ENGINEERING MATHEMATICS
FINAL EXAM REVIEW PROBLEMS

May 22, 2018



Name: _____

Be sure to show your work!

1. Specify the type of the differential equation : $(1 + x^2)dy = (x + xy^2)dx$ and find the general solution. Show your work in detail.

2. Specify the type of the differential equation : $(\sin x)y' + (\cos x)y = \ln x$ and find the general solution. Show your work in detail.

3. Make a suitable change of variable and find the general solution of $\frac{dy}{dx} = \frac{x + y + 1}{x + y - 1}$.

4. Specify the type of the differential equation : $x \frac{dy}{dx} + y = x^2y^2$ and find the general solution. Show your work in detail.

5. Determine whether or not the differential equation : $(e^x \sin y - 2y \sin x) + (e^x \cos y + 2 \cos x)y' = 0$ is exact. Solve it.

6. Specify the type of the differential equation $y' = 1 + (y/x) + (y/x)^2$ and find the general solution. Show your work in detail.

7. Solve initial value problem

$$y' = \frac{yx^5}{2 + x^6}$$
$$y(1) = 1$$

8. Consider the differential equation $(2x + 3) + (2y - 2)y' = 0$. Determine whether this equation is exact or not. If it is, solve it.

9. Consider the differential equation $(ye^{2xy} + x) + axe^{2xy}y' = 0$. Determine for which value of a this equation is exact, and then solve it with this value of a .

10. Consider the differential equation $(ye^{2xy} + x) + axe^{2xy}y' = 0$. Determine for which value of a this equation is exact, and then solve it with this value of a .

11. Consider the differential equation $(2xy - y^3) + (x^2 - 3xy^2)y' = 0$. Determine whether this equation is exact or not. If it is, solve it.

12. Specify the type of the differential equation : $\frac{dy}{dx} + xy = xe^{-x^2}y^{-3}$ and find the general solution. Show your work in detail.

13. Specify the type of the differential equation :

$$y' = y(1 - y)$$
$$y(0) = 1/2$$

and find the general solution. Show your work in detail.

14. Specify the type of the differential equation : $y' + 2y = \cos(3x)$ and find the general solution. Show your work in detail.

15. On what interval we expect unique solutions to

$$y' = \frac{y^2}{1 - x^2}$$
$$y(0) = 0$$

Show your work in detail.

16. Specify the type of the differential equation : $y' = xe^{x^2 - \ln y^2}$ and find the general solution. Show your work in detail.

17. Find the differential equation of all circles of radius 1

18. Show that the function

1. $y = C_1 \sin(\pi x) + C_2 \cos(\pi x)$, where C_1 and C_2 are constants, is a solution of the differential equation:
 $y'' + \pi^2 y = 0$.

2. Solve the initial value problem:

$$y'' + \pi^2 y = 0, \quad y(0) = 1, \quad y'(0) = -1$$

19. Solve $y'' - y = 2e^x$.

20. Solve $y'' - 2y' - 3y = 3x^2 - 5$.

21. Solve $y'' + y' - 12y = e^x + e^{2x} - 1$; $y(0) = 1$, $y'(0) = 3$.

22. Find the general solution of the differential equation $y'' - 2y' + y = \frac{e^x}{x}$.

23. Find the general solution of the differential equation $(1 - x^2)dy + xydx = xy^2dx$.

24. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$.

25. Find the general solution of the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.

26. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{x^2 - y^2} + y}{x}$.

27. Find the general solution of the differential equation $x \frac{dy}{dx} = y + x\sqrt{x^2 + y^2}$

28. Find the general solution of the differential equation $y'' - 2y' + y = xe^x$.

29. Find the general solution of the differential equation $y'' + 5y' + 6y = e^{2x}$.

30. Find the general solution of the differential equation $y''' + 2y'' + y = e^{2x} + \cos 2x$.

31. Find the general solution of the differential equation $y'' + 4y = 5 + \sin 2x$.

32. Find the general solution of the differential equation $y'' + y = x \sin x$.

33. Find the general solution of the differential equation $y'' + 4y = \sin^2 x$.

34. Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = x^3$.

35. Find the general solution of the differential equation $y''' - y' = 4 \cos(2x)$.

36. Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = x^3$.

37. Find the general solution of the differential equation $9y'' + 12y' + 4y = 18e^{x/3} + 25 \cos\left(\frac{x}{3}\right)$.

38. Find the general solution of the differential equation $y'' - 2y' + y = \frac{e^x}{x^2}$.

39. Find the general solution of the differential equation $y'' - 6y' + 8y = 0$, $y(0) = 3$, $y'(0) = -2$.

40. Find a second order linear equation with constant coefficients whose general solution is: $c_1e^{2x} + c_2e^{-5x}$

41. Find the general solution of the differential equation $3y'' + y' - 2y = 2 \cos x$.

42. Find the general solution of the differential equation $y^{(5)} - 3y''' - 2y'' = 0$.

43. Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = x^3$.

44. Find the general solution of the differential equation $x^2y' = xy + y^5$.

45. Find the general solution of the differential equation $y''' - 3y'' + 3y' - y = 4e^x$.

46. Let $y_1 = x$ be a solution of the second order linear equation $x^2y'' - xy' + y = 0$. Find a second, linearly independent solution.

47. Find the general solution of the differential equation $y'' - 4y' + 4y = 6xe^{2x}$.

48. Solve the initial value problem $(2y - \sin y)y' = \sin x - x$, $y(0) = 0$.

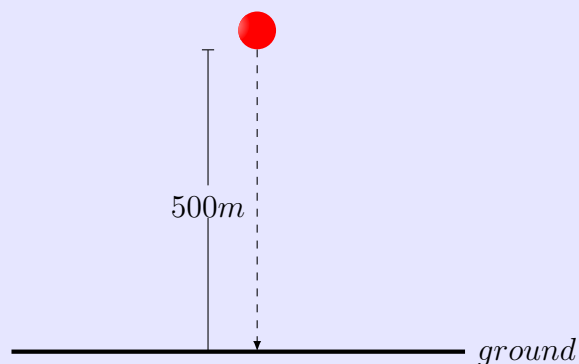
49.

1. Solve the initial value problem $y'' - 4y' + 3y = 0$, $y(0) = -1$, $y'(0) = 1$.

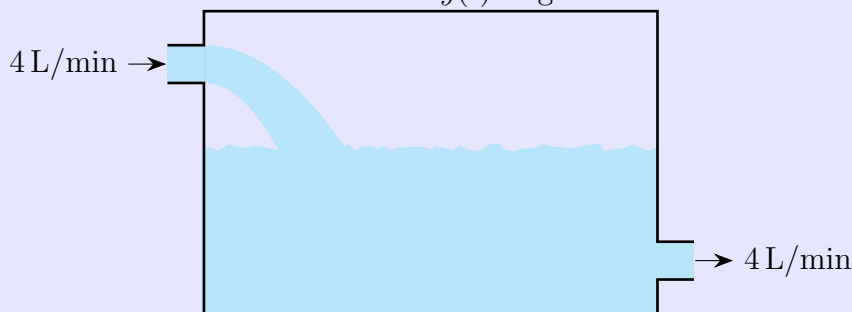
2. Find $\lim_{x \rightarrow -\infty} y(x)$.

50. Solve the initial value problem $4y'' - y = 0$, $y(0) = 2$, $y'(0) = \lambda$.

51. An object is dropped from a height of $500m$. When will the object reach ground level, and with what speed?



52. A tank contains 200 liters of fluid in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is then pumped into the tank at a rate of 4 liters/min, the well-mixed solution is pumped out at the same rate. Find the number $y(t)$ of grams of salt in the tank at time t .



53. Let us assume that you took out college loans totaling 60000.00 with interest of 7.5%. You have an online payment plan which continuously deducts money from your bank account at a rate which comes out to 15000.00 per year. How long will it take you to pay off the loan?

54. Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y} + x^2 e^y$.

55. Find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}}$.

56. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{x^2 - y^2} + y}{x}$.

57. Find the general solution of the differential equation $y^2 \frac{dy}{dx} = x + y^3$.

58. Find the general solution of the differential equation $(y + \cos x + \sin y + y)dx + (\sin x + x \cos y + x)dy = 0$.

60. Find the general solution of the following differential equation $\frac{dy}{dx} = 3x^2(1 + y^2)$.

61. Find the general solution of the following differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + 2y^2}$.

62. Use the method of variation of parameters to find the general solution to each of the following equations.

(a) $y'' - 2y' + y = 4e^x$

(b) $y'' - 2y' + 2y = 4e^x \sin x$

(c) $y'' - 4y' + 4y = xe^{2x}$

63. Specify the type of the differential equation : $x \frac{dy}{dx} - 5y = 3xy^{7/3}$ and find the general solution. Show your work in detail.

64. Specify the type of the differential equation : $\frac{dy}{dx} = \frac{e^{x-y}}{1+e^x}$, $y(1) = 0$ and find the general solution. Show your work in detail.

65. Specify the type of the differential equation : $y'''(x) + 3y''(x) + 2y'(x) = 0$ and find the general solution. Show your work in detail.

66. Specify the type of the differential equation : $y^{(4)}(x) - 8y''(x) + 16y = 0$ and find the general solution. Show your work in detail.

67. Specify the type of the differential equation : $2x^2y - x^3 \frac{dy}{dx} = y^3$ and find the general solution. Show your work in detail.

68. Specify the type of the differential equation : $e^y + y \cos(x) + (xe^y + \sin(x) + e^y) \frac{dy}{dx} = 0$ and find the general solution. Show your work in detail.

69. Specify the type of the differential equation : $x^2 \frac{dy}{dx} = y^2 + xy - x^2$ and find the general solution. Show your work in detail.

70. Find a differential equation whose general solution is $y = c_1e^{2x} + c_2e^{-3x}$.

71. Solve the initial value problem

$$y'' - y' - 2y = 0$$

$$y(0) = \alpha$$

$$y'(0) = 2$$

and then find α so that the solution approaches zero as $t \rightarrow \infty$.

72.

73.

74.

75.

76.

77.