Instructions: Multiple Choice Questions (65 points)

Q1 Evaluate $\int_0^{\frac{\pi}{2}}\int_0^1 \frac{\cos y}{\sqrt{1-x^2}}dx\,dy$

(A) $\frac{2\pi}{3}$ (C) $\frac{\pi}{2}(\frac{e+1}{2e})$ (E) None

(B) $\pi/8e$ (D) $\pi/2$

Q2 Evaluate $\int_1^2\int_0^\pi x\cos(xy)dydx$

(A) $\ln\sqrt{61}$ (C) -8π (E) None

(B) $\frac{-2}{\pi}$ (D) $\sqrt{2}\pi$

Q3 Find the angle between the vectors: $\mathbf{u} = \sqrt{2}\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $\mathbf{v} = \sqrt{2}\mathbf{i} - \mathbf{k}$.

(A) π (C) $\pi/6$ (E) None

(B) $\pi/3$ (D) $1+\pi$

Q4 Find the directional derivative of the function $f(x,y) = x^3 + y^2$ at $(0,1)$ in the direction of $\overrightarrow{v} = (1,1)$.

(A) 2 (C) $1/\sqrt{3}$ (E) None

(B) $\frac{-1}{5}$ (D) $\frac{2}{5}$

Q5 Find the center and the radius of the sphere

$$x^2 + y^2 + z^2 - 2y - 4z - 4 = 0$$

(A) $(-1, 3/2, -2)$, $R = 7/2$. (C) $(1, -3/2, 2)$, $R = 5$.

(B) $(0, 1, 2)$, $R = 3$. (D) None

Q6 What is the length of the arc described $r(t) = (\cos t, \sin t, 5t)$ where $\pi \le t \le 6\pi$

(A) $5\pi\sqrt{26}$ (C) $8\sqrt{6}$ (E) None

(B) $2\pi\sqrt{61}$ (D) $4\pi\sqrt{61}$

Q7 Compute $\int_0^6 \sqrt{6x - x^2}dx$

(A) π (C) 4π (E) None

(B) $2\pi^2$ (D) $\frac{9\pi}{2}$

Q8 Find the equation of the plane through the point $(1,2,-3)$ and perpendicular to the vector $(2,1,-3)$

(A) $2x + y - 3z = -8.$ (C) $2x + y + 3z = -18.$

(B) $2x + y - 3z = 13.$ (D) None

Q9 The value of $\lim_{(x,y)\to(0,0)} \frac{x^2y^3 + x^3y^2 - 5}{2 - xy + \cos(xy)}$

(A) -2 (C) $-5/3$ (E) None

(B) 1 (D) DNE

Q10 Let $\mathbf{v} = (2,-1,3)$. Which of the following vectors is perpendicular to $\mathbf{v}$: $\mathbf{a} = (4,-2,6)$ $\mathbf{b} = (2,4,0)$ $\mathbf{c} = (1,1,1)$

(A) **a** (C) **c** (E) None

(B) **b** (D) **a, b**

Q11 Find $\lim_{t\to 0}\left[\frac{\sin t}{t}\overrightarrow{i} + (1+t)\overrightarrow{j} + (t-1)^4\overrightarrow{k}\right]$

(A) $\overrightarrow{i}$ (C) $\overrightarrow{i} - \overrightarrow{k}$ (E) None

(B) $\overrightarrow{i} + \overrightarrow{j}$ (D) $\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$

Q12 Find the sum of the series $\sum_{n=2}^{\infty}\frac{3^n - 2^n}{6^n}$

(A) $\frac{1}{3}$ (C) 12π (E) None

(B) $\frac{25}{6}$ (D) $\frac{11}{7}$

Q13 Let $z = \cos(x - y)$ for a differentiable function. Then $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$ is

(A) 1 (C) $\sqrt{2}$ (E) None

(B) 2 (D) 0

True and False(10pts)

Q1 The surfaces $x^2 - y^2 + z^2 = 2$ and $x^2 + y^2 + z^2 = 2$ have the same tangent plane at (1, 0, 1).

T F

Q2 $(0, 0)$ is a global maximum of the function $f(x, y) = 6 - x^2 - y^2$.

T F

Q3 If two functions $f(x, y)$ and $g(x, y)$ have the same critical points, then function $f(x, y) = \lambda g(x, y)$.

T F

Q4 If $|\overrightarrow{v} \times \overline{w}| = (0, 0, 0)$ then $\overrightarrow{v} = \overrightarrow{w}$

T F

Q5 The $\displaystyle\int_1^{\infty} \frac{1}{x^4} dx$ is divergent.

T F

Classical Problems. Show all your work. No work=No credit!

Q1(15pts) Evaluate $\displaystyle\int_0^2 \int_{y^2}^4 \cos\left(x^{3/2}\right) dxdy$.

Solution:

continued --→

Q2(15pts) Use Lagrange multipliers to find the maximum value of the function $f(x, y, z) = x + 2y - 2z$ on the sphere $x^2 + y^2 + z^2 = 9$.

Solution: