Sample Final Exam Review Problems

☞ 75 min

S1. Which of the following is the solution to the differential equation

$$\frac{dy}{dx} = \frac{x}{y}$$

with the initial condition $y(0) = 1$.

(A) $y^2 - x^2 = 1$

(B) $y = 1/(x+1)$

(C) $y = x + 1$

(D) $y^2 - x = 1$

(E) None of the above

S2. The differential equation $2\frac{dy}{dx} + x^2 y = 2x + 3, \quad y(0) = 5$ is

(A) linear

(B) nonlinear

(C) Bernoulli

(D) exact

(E) None of the above

S3. A differential equation is considered to be ordinary if it has

(A) one dependent variable

(B) more than one dependent variable

(C) one independent variable

(D) more than one independent variable

(E) None of the above

S4. Classify the following differential equation:

$$e^x \frac{dy}{dx} + 3y = x^2 y$$

(A) Separable and not linear

(B) Linear and not separable.

(C) Both separable and linear.

(D) Neither separable nor linear.

(E) None of the above

S5. Consider the linear differential equation:

$$\frac{dy}{dx} + \frac{1}{1+x} y = 1 + x.$$

The integrating factor is

(A) e^x

(B) $1 + x$

(C) xe^x

(D) $x - xe^x$

(E) None of the above

S6. If $y = e^{2x}$ is a solution to

$$y'' - 5y' + ky = 0,$$

what is the value of k ?

(A) 6

(B) 9

(C) -1

(D) $1/3$

(E) None of the above

S7. Find a particular solution to

$$y'' + 2y' + y = x.$$

(A) $y_p = x - 2$

(B) $y_p = x$

(C) $y_p = 2x - 1$

(D) $y_p = 2x - 2$

(E) None of the above

S8. Find the general solution to

$$y'' - 4y' + 8y = 0$$

(A) $y = Ae^x + Be^{2x}$

(B) $y = e^{2x}(A\cos(2x) + B\sin(2x))$

(C) $y = e^{-2x}(A\cos(4x) + B\sin(4x))$

(D) $y = Ae^x + Bxe^{2x}$

(E) None of the above

S9. Find the solution to the initial value problem:

$$y'' + 3y' + 2y = 0, \quad y(0) = 0, \qquad y'(0) = -2$$

(A) $y = 2e^{-2x} - 2e^{-x}$,

(B) $y = e^{-2x} - e^{-x}$,

(C) $y = e^{-2x}\sin(4x)$,

(D) $y = 2e^x - 2e^{2x}$,

(E) None of the above

S10. Classify the following differential equation:

$$xy''' - (y')^4 + y = 0$$

(A) 3rd-order, linear,

(B) 3rd-order, non-linear,

(C) 4th-order, linear,

(D) 4th-order, non-linear ,

(E) None of the above

S11. For which values of m is the function $y = x^m$ a solution of the differential equation

$$x^2y'' - 5xy' + 8y = 0?$$

(A) m = 2, 3,

(B) m = 2, 4,

(C) m = 3, 4,

(D) m = -2, -4 ,

(E) None of the above

S12. Which of the following differential equations are equivalent to

$$\frac{d}{dx}(e^x y) = xe^x$$

(A)
$$e^x \frac{dy}{dx} = xe^x$$

(B)
$$\frac{dy}{dx} + e^x y = xe^x$$

(C)
$$\frac{dy}{dx} = xe^x$$

(D)
$$\frac{dy}{dx} = x - y$$

(E) None of the above

S13. An integrating factor that makes the differential equation

$$ydx - xdy = 0$$

exact is

(A)
$$y$$

(B)
$$1/x$$

(C)
$$1/y^2$$

(D)
$$xy$$

(E) None of the above.

S14. Which of the following differential equations is exact?

(A)
$$2xydx + (x^2 + 3y^2)dy = 0$$

(B)
$$e^x dx + xe^y dy = 0$$

(C)
$$y' = ln(xy)$$

(D)
$$ydx - xdy = 0$$

(E) None of the above

S15. The general solution to

$$\frac{dy}{dx} = y^2$$

has the form

(A)
$$y = \frac{c}{x+1}$$

(B)
$$y = \frac{1}{x+c}$$

(C)
$$y = \frac{-1}{x+c}$$

(D)
$$y = \frac{c}{x-1}$$

(E) None of the above

S16. Which of the following is a linear ordinary differential equation

(A)
$$yy'' = 1$$

(B)
$$y'' - y = 1$$

(C)
$$yy'' - \frac{1}{y} = y$$

(D)
$$y' = \sin(y)$$

(E) None of the above

S17. Which of the following differential equations is not separable?

(A) $$y' = xy$$

(B) $$e^{x+y}y' = 2x$$

(C) $$y + y' = e^x - e^y$$

(D) $$y' = ye^x$$

(E) None of the above

S18. The Wronskian of

$$f_1(x) = x^2 \sin x \quad \text{and} \quad f_2(x) = x^2 \cos x$$

is

(A) $$x^2$$

(B) $$-x^2$$

(C) $$x^4$$

(D) $$-x^4$$

(E) None of the above

S19. The degree of the differential equation

$$(y'')^2 + 3(y')^3 + 2y^4 = 0$$

is

(A) 3,

(B) 2,

(C) 1,

(D) 4,

(E) None of the above

S20. Find the solution to the initial value problem:

$$\frac{dy}{dx} = e^{-x^2}, \quad y(3) = 5$$

(A) $$y = 5 + \int_3^x e^{-t^2}dt$$

(B) $$y = 3 + \int_3^x e^{-t^2}dt$$

(C) $$y = -5 + \int_0^x e^{-t}dt$$

(D) $$y = \int_5^x e^{-t^2}dt$$

(E) None of the above.

S21. The general solution to

$$y^4dx + 4xy^3dy = 0$$

is given by

(A) $$y = cx$$

(B) $$y = cx^2$$

(C) $$y = cx^3$$

(D) $$y = cx^4$$

(E) None of the above.

S22. For the 1st-order linear differential equation

$$xy' + 4y = x^2$$

what is the integrating factor?

(A) $$\mu = e^x$$

(B) $$\mu = e^x$$

(C) $$\mu = e^x$$

(D) $$u = e^x$$

(E) None of the above.

S23. Consider the 2nd-order non-homogeneous differential equation

$$y'' - 4y' + 3y = e^x + x^2$$

What is the general solution of homogeneous part?

(A) $$y_h = c_1e^x + c_2x$$

(B) $$y_h = c_1e^x + c_2x$$

(C) $$y_h = c_1e^x + c_2x$$

(D) $$y_h = c_1e^x + c_2x$$

(E) None of the above.

S23. Which of the following is NOT a fundamental set of solutions for

$$y'' - y = 0?$$

What is the general solution of homogeneous part?

(A)
$$(e^x, e^{-x}$$

(B)
$$(e^x, e^{-x}$$

(C)
$$(e^x, e^{-x}$$

(D)
$$(xe^x, e^{-x}$$

(E) None of the above.

S24. The function $y_1 = e^x$ is a solution to

$$(x-1)y'' - 2xy' + (x+1)y = 0, \quad x > 1$$

Use reduction of order to find a function y_2 so that the pair y_1, y_2 form a fundamental set of solutions to the differential equation.

S25. Find the general solution of

$$x^2y'' + xy' - y = x^2 + 1,$$

given that $y_1 = x$ is a solution of the complementary equation $x^2y'' + xy' - y = 0$.

S26. Find the general solution and a fundamental set of solutions of

$$x^2y'' - 3xy' + 3y = 0,$$

given that $y_1 = x$ is a solution.

S27. Find a particular solution of

$$y'' + 3y' + 2y = \frac{1}{1+e^x}.$$

Then find the general solution.

S28. Solve the initial value problem

$$(x^2-1)y'' + 4xy' + 2y = \frac{2}{x+1}, \quad y(0) = -1, \quad y'(0) = -5,$$

given that, $y_1 = \frac{1}{x-1}$ and $y_2 = \frac{1}{x+1}$ are solutions of the equation $(x^2-1)y'' + 4xy' + 2y = 0$.

S29. Find the solution of the differential equation:

$$y' = \frac{x+1}{e^y}, \quad y(0) = 2.$$

S30. Solve by the variation of parameters:

$$y'' + 4y = 4\tan x.$$

Ans:$y = c_1\cos 2x + c_2\sin 2x - \cos 2x(\ln(\sec 2x + \tan 2x))$

S31. Solve by the variation of parameters:

$$y'' + y = x\sin x.$$

Ans:$y = c_1\cos x + c_2\sin x + \frac{x}{4}\sin x - \frac{x^2}{4}$

S32. Solve the following equation,

$$(y')^2 + y' - 6 = 0.$$

S33. Solve the following equation,

$$(y')^2 + (x+y)y' + xy = 0.$$

S34. Solve the following equation,

$$(1+e^{x/y})dx + e^{x/y}(1 - \frac{x}{y})dy = 0.$$

S35. Solve the following equation,

$$x^2y'' + 2xy' - 6y = 0,$$

by guessing a solution of the form $y = x^m$ and determining the value of m.

S36. Solve the following equation,

$$y'' + 10y' + 28y = 0.$$

S37. Use the solution $y_1 = e^x$ and reduction of order to find second solution y_2 to

$$y'' + 3y' + 2y = 0.$$

S38. Find the general solution to

$$y'' - \frac{1}{x}(y' + x^2\cos x) = 0.$$

S39. Use the solution $y_1 = x$ and reduction of order to find second solution y_2 to

$$x^2y'' + 2xy' - 2y = 0.$$

S40. Find general solutions of

$$y''' + 4y'' - 7y' - 10y = 0, \quad y(0) = -3, y'(0) = 12, y''(0) = -36$$