Instructions: Multiple Choice Questions (60 points)

Q1 Compute $\int_0^4 \sqrt{4x - x^2}\,dx$

(A) 2π (B) $2\pi^2$ (C) 4π (D) None

Q2 Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{3^n - 2^n}{6^n}$

(A) $\frac{25}{6}$ (B) $\frac{1}{3}$ (C) 12π (D) None

Q3 Which of the following statements are true about the series $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$.

1. The series diverges by the Ratio Test.

2. The series diverges by the Test for Divergence.

3. The series diverges by comparison to the harmonic series.

(A) 1 only   (B) 2 only   (C) 3 only   (D) None

Q4 Find the angle between the vectors: $\mathbf{u} = \mathbf{i} - \mathbf{k}$ and $\mathbf{v} = \mathbf{j} + \mathbf{k}$.

(A) π (B) $2\pi/3$ (C) $\pi/6$ (D) None

Q5 Let, $x^2 + y^2 + z^2 - 4y - 8z + 4 = 0$. Find the center and the radius of the sphere.

(A) $(1,1,2)$, $R = 1$.   (B) $(1,2,2)$, $R = 5$.   (C) $(0,2,4)$, $R = 4$.   (D) None

Q6 Let $\mathbf{v} = (2, -1, 3)$. Which of the following vectors is perpendicular to $\mathbf{v}$: $\mathbf{a} = (4, -2, 6)$ $\mathbf{b} = (2, 4, 0)$ $\mathbf{c} = (1, 1, 1)$

(A) $\mathbf{a}$ (B) $\mathbf{b}$ (C) $\mathbf{c}$ (D) None

Q7 Find the directional derivative of the function $f(x,y) = \sin(x^2 + y^2)$ at $(1,1)$ in the direction of $\vec{v} = (1,1)$.

(A) $2\sqrt{2}\cos 2$ (B) $1/\sqrt{3}$ (C) $\sqrt{2}\sin 2$ (D) None

Q8 What is the length of the arc described $r(t) = (t^2 + 5, \frac{4t^{3/2}}{3}, t - 6)$ where $0 \le t \le 1$

(A) 1   (B) 8   (C) 2   (D) None

Q9 The value of $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y^3 + x^3 y^2 - 5}{2 - xy + \cos(xy)}$

(A) $-5/3$ (B) DNE (C) 1   (D) None

Q10 Let $z = \cos(x - y)$ for a differentiable function. Then $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ is

(A) 1   (B) 2   (C) 0   (D) None

Q11 Evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \int_0^4 \cos y \sqrt{4x - x^2}\,dx\,dy$

(A) $\pi\sqrt{3}$ (B) $\frac{2\pi}{3}$ (C) $\pi/8e$ (D) None

Q12 Evaluate $\displaystyle\int_1^2 \int_0^\pi y \sin(xy)\,dx\,dy$

(A) $\frac{2}{\pi}$ (B) 1   (C) $-\pi$ (D) None

Classical Problems. Show all your work. No work=No credit!

Q13(20pts) Evaluate $\int_0^2 \int_{y^2}^4 e^{x^{3/2}} \, dx \, dy$.

Solution:

Q14(20pts) Use Lagrange multipliers to find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $x + 2y + 3z = 4$.

Solution: