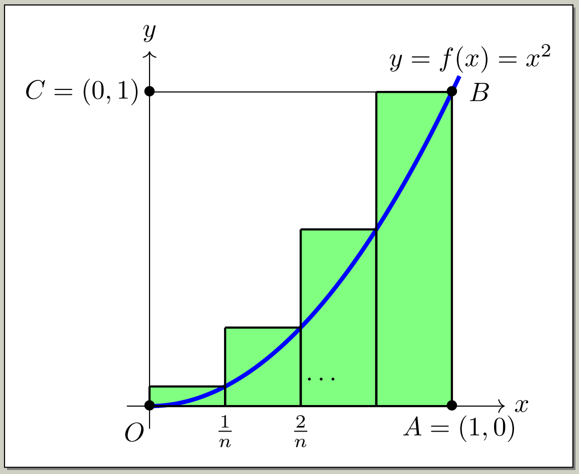
Math Problems of the Month

Q1. Show that the area D of the region OAB bounded by the parabola $y = x^2$, the x-axis and the line $x = 1$ is one-third that of the square $OABC$. (Hint: Approximate the area D under the curve $y = x^2$ on $[0, 1]$ using upper rectangles.)



Q2. Let m, n be strictly positive integers. Prove that

$$\lim_{x\to 1} \frac{x^m - 1}{x^n - 1} = \frac{m}{n}.$$

Q3. Evaluate the limit :

$$\lim_{x\to 0} \sqrt{\sqrt{x} + x^3}\cos\left(\frac{\pi}{x}\right).$$

Q4. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Prove that $\lim_{x\to 0} f(x) = 0$.

Q5. A six-foot tall man is walking away from a fifteen-foot high lamppost.

(i) Find a formula for the length y of his shadow as a function of his distance x from the lamppost.

(ii) What is the rate of change of y with respect to x?

Q6. A right triangle has base $|BC| = b$ and height $|AC| = h_1$. An infinite sequence of nested right triangles is constructed inside the triangle by dropping perpendiculars as shown in the diagram.

(i) Find an expression for the edge h_n.

(ii) What is $\lim_{n\to\infty} h_n$.

(iii) Find an expression for the sum $s_n = h_1 + h_2 + \ldots h_n$.

(iv) What should be $\sum^{\infty} h_n$

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