Math Problems of the Month

Q1 Verify Rolle's theorem for the function

$$f(x) = (x-a)^m (x-b)^n$$

in the interval $a \leq x \leq b$, where m and n are positive integers.

Q2 If $f : [-5, 5] \to \mathbb{R}$ is differentiable function and $f'(x)$ does not vanish anywhere then prove that $f(-5) \neq f(5)$.

Q3 If $\frac{a_n}{n+1} + \frac{a_{n-1}}{n} + \frac{a_{n-2}}{n-1} + \ldots + \frac{a_1}{2} + a_0$, then show that the equation

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 = 0$$

has at least one root in $(0, 1)$.

Q4 When traveling $x\,km/hour$, a truck uses fuel at the rate of $\frac{1}{400}(\frac{3600}{x} + x)\,lt/km$ (liters per km). The fuel price is $4000TL$ per liter. What should be the speed of the truck to cover a $600\,km$ distance with minimum fuel consumption.

Q5 When traveling $x\,km/hour$, a truck uses fuel at the rate of $\frac{1}{400}(\frac{3600}{x} + x)\,lt/km$ (liters per km). The fuel price is $4000TL$ per liter and and driver is paid 15 TL per hour. What should be the speed of the truck to cover a $600\,km$ distance with minimum fuel consumption.

Q6 A man $2m$ high walks at a uniform speed of 6km/hour away from a lamp post $6m$ high. Find the rate at which the length of his shadow increases.

Q7 A truck is to be driven 130 km at a constant speed of x km/hr. Speed laws require that $40 \leq x \leq 120$. Assume that gasoline costs 40 TL/liter and is consumed at the rate of $2 + \frac{x^2}{360}$ liter /hours. If the driver is paid 150 TL /hours, find the most economical speed and the total cost for the trip.

JANUARY