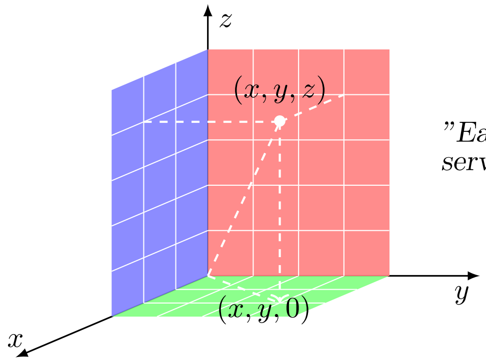
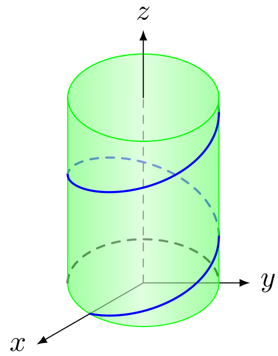


"Each problem that I solved became a rule, which served afterwards to solve other problems."

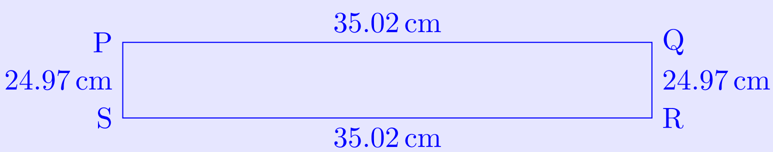
Rene Descartes



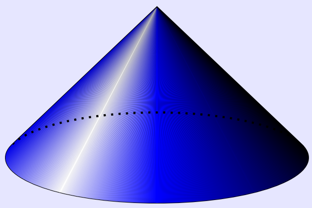
Q1 A bee is flying around a room in which the temperature is given by $T(x, y, z) = x^2 + y^4 + 2z^2$. The bee is at the point $(1, 1, 1)$ and realizes that he's cold.

1. In what direction should he bee to warm up most quickly?
2. If bee lies in this direction, what will be the instantaneous rate of change of his temperature?

Q2 Approximate the area of a rectangle of dimensions 35.02 by 24.97 units.



Q3 The altitude of a right circular cone is 15 inches and is increasing at $0.2in/min$. The radius of the base is 10 inches and is decreasing at $0.3in/min$. How fast is the volume changing?



Q4 Find the equation of a plane which is tangent to the ellipsoid $x^2 + y^2 + 2z^2 = 4$ at the point $(1, 1, 1)$.

Q5 If $z = T(x, y) = x^2 + 3xy^4$ represents the temperature at the point (x, y),then the composite function $z = T(\cos t, \sin t)$ represents the temperature at points on the curve $C =: x = \cos t, y = \sin t$. Find $\frac{dz}{dt}$ when $t = 0$ and explain it.

Q6 Calculate $\displaystyle\int_0^2 \int_x^2 e^{-y^2} dydx$.

Q7 Find the equation of the tangent plane to the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1$ at the point $P = (x_0, y_0, z_0)$.

Q8 Find the minimum value of the distance from the origin to the plane $ax + by + cz + d = 0$.

Q9 The electrical potential V at any point (x, y) is given by $V = \ln(x^2 + y^2)$. Find the rate of change of V at the point $(3, 4)$ in the direction toward the point $(2, 6)$.

Q10 Find the points on $z = \sqrt{xy} - 2$ nearest the origin.

Q11 The dimensions of a rectangular box are measured to be $75cm$, $60cm$, and $40cm$, and each measurement is correct to within $0.2cm$. Use differentials to estimate the largest possible error when the volume of the box is calculated from these measurements.

Q12 If $z = f(x, y) = x^2 + 3xy^4$ where $x = \cos t$ and $y = \sin t$. Find $\frac{dz}{dt}$ when $t = 0$ and explain it geometrically.

Q13 Which cylindrical soda cans of height h and radius r has minimal surface for fixed volume?

Q14 Does the function $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2$ have a global maximum or a global minimum?

Q15 A copper wire is bent into a square shape with vertices $(0, 0), (2, 0), (2, 2)$ and $(0, 2)$. The wire is heated so that the temperature at any point is given by $T(x, y) = 5xy - x^2 + y^2 - 5x - 2y + 4$. Find the temperature at the hottest points on the wire.

Q16 Find the local maximum, local minimum and saddle points of $z = x^2 - 6x - 8y + y^2$.

Q17 Find the absolute maximum and absolute minimum of $z = x^2 - 6x - 8y + y^2$ over the region $0 \leq x \leq 5$ and $0 \leq y \leq 25$.

Q18 Describe the intersection of the graphs of $x^2 + y^2 = 1$ and $z = 4$.

Q19 If z is a function of x and y then find $xz_x + yz_y$.

Q20 If $z = \ln(x^2 + y^2)$ then find $xz_x + y_z y$.

Q21 If $u = x^2 - y^2 + z^2$ and $x = \sin t, \quad y = e^t \quad z = t$ then find $\frac{du}{dt}$.

Q22 Find an equation of the tangent plane to $z = xy$ at $(2, \frac{1}{2}, 1)$.

Q23 Find the extrema of $f(x, y) = xy + 14$ subject to $x^2 + y^2 = 1$

Q24 Find the directional derivative of $f(x, y) = xy^2$ at $(1, 3)$ in the direction toward $(4, 5)$.

Q25 Find positive numbers x, y, z such that $x + y + z = 18$ and xyz is a maximum.

Q26 Use polar coordinates to evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dydx$

Q27 Find the critical point(s) of $f(x, y) = x^3 + y^3 - 3xy$.

Q28 Find the limit $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + y^2}$ along the lines $y = 0$ and $y = x$.

Q29 Find the limit $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2}$ along the line through origin and $y = x^2$.

Q31 Find the volume of the region below $f(x, y) = xy$ and over the region $R = \{(x, y) : 0 \le x \le 1, x \le y \le 1\}$

Q32 Find the volume of the solid between $f(x, y) = x^2 + y^2$ and $f(x, y) = x^2 - y^2$ over the region $R : y = 0, x = y^2, \quad y = 1, x = y$.

Q33 Evaluate the iterated integral $\displaystyle\int_0^1 \int_{|y|}^1 \cos(x^2) dxdy$

Q34 Evaluate the iterated integral $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2) dydx$

Q35 Use double integral and compute the area of the region enclosed by the cardioid $r = 1 + cos(\theta)$ where $\theta \in [0, 2\pi]$

Q36 In what direction the function $f(x, y) = x^3 y$ increasing the fastest at the point $(2, 3)$.

Q37 Find the minimum value of the function $x + y + z$ subject to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Q38 Don't forget to solve old exam questions and additional problems on my Calculus-II website.