ISTANBUL COMMERCE UNIVERSITY

Electrical-Electronics Engineering
EEE202 Electro-technic Laboratory

Theory

Part 2

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1 CIRCUIT THEOREMS

Number of sources and loads can be in 4 types in electrical circuits;

1. Single source, single load,
2. Single source, multiple load,
3. Multiple source, single load,
4. Multiple source, multiple load.

In electrical circuits, the sources and the loads can be connected in series, parallel or mixed. Ohm and Kirchhoff laws are valid in all the solution methods of electrical circuits. There are some practical methods derived from Ohm's and Kirchhoff's laws in the analysis of the electrical circuits. These methods are called the "circuit theorems". The circuit theorems are used in solutions of the direct current (DC) circuits and the alternative current circuits (AC). The most common ones of the circuit theorems are as follows;

1. Superposition theorem,
2. Thevenin theorem,
3. Norton theorem.

The theorems mentioned above will be discussed respectively.

1.1 Superposition Theorem

The superposition theorem is used in order to find the voltage at a junction point when the circuit has multiple sources. While analyzing the circuit, in each step only one of the sources is considered and the rest of the sources are killed (voltage sources are short-circuited and the current sources are open circuited). Let's determine the voltage of the junction A in Figure 1 with respect to the ground.

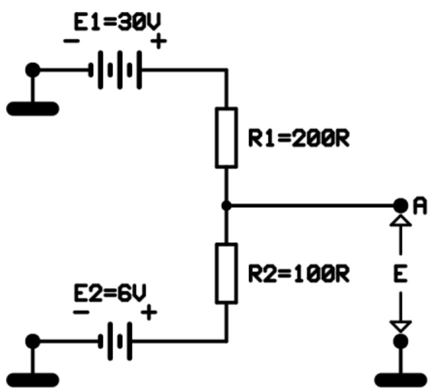

Figure 1: The junction A is determined on circuit that have two sources.

The circuit in Figure 1 is basically a voltage divider. Firstly, effect of the E_1 source is found on the circuit. For this purpose, the E_2 source is removed from the circuit and its terminals are short-circuited.

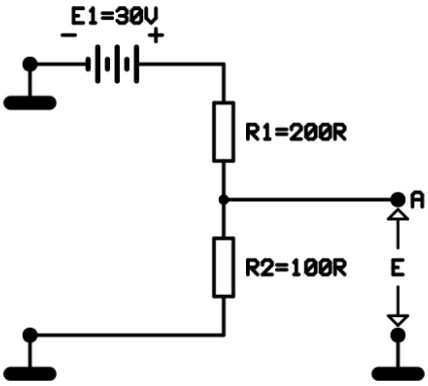

Figure 2: E_2 source is removed from the circuit to find effect of E_1.

Considering Figure 2 , the voltage of junction A is the voltage between the terminals of R_2. Because this voltage is created by E_1, we call this voltage E_{1A} and let's find the value using voltage division method.

$$
\begin{aligned}
E_{1A} &= \frac{R_2}{(R_1 + R_2)} \times E_1 \\
&= \frac{100}{200 + 100} \times 20 \\
&= 10\ Volt
\end{aligned}
$$

Now, E_1 source is removed in order to find the effect of E_2 on the circuit. To remove E_1, its terminals are short-circuited and E_1 is removed from the circuit.

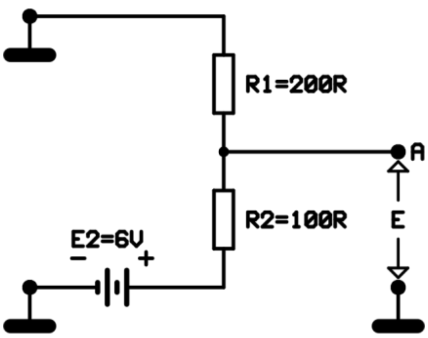

Figure 3: Terminals of E_1 are short-circuit to find effect of E_2.

The voltage at junction A is called "E_{2A}" as it is created by E_2. Let's determine this voltage using voltage division method.

$$E_{2A} = \frac{R_1}{R_1 + R_2} \times E_2$$
$$= \frac{200}{100 + 200} \times 6$$
$$= 4\ Volt$$

Superposition states that the voltage of the point A with respect to the ground is the sum of these two voltages. Let's denote total voltage as "E_P", and we can find;

$$E_P = E_{A1} + E_{A2}$$
$$= 10 + 4$$
$$= 14\ Volt$$

In this circuit, two sources are connected in order to increase voltage drop on the load. If the polarities of one of the sources are reversed, then the voltage on the load is decreased.

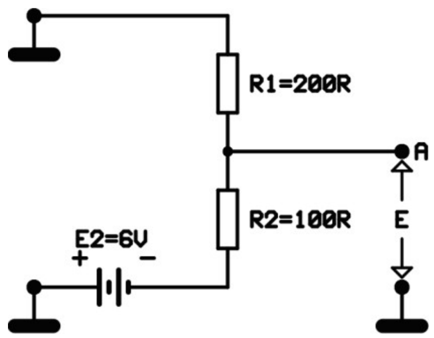

Figure 4: Polarities of E_2 are reversed to decrease total voltage on Junction A.

In Figure 4, the polarities of E_2 are reversed. There is no change in the analysis of E_{1A} because E_2 is shorted to find E_{1A}. As we did before, let's remove E_1 in order to find the effect of E_2 on the circuit. Let's short circuit the terminals of the source. This situation can be seen in Figure 4. If we connect a digital voltmeter whose − pin is on the ground and the + pin is on point A, we can see that the measured voltage will be negative. Therefore the circuit is called negative feedback voltage divider. The voltage at the A junction is called "E_{2A}" as it is created by the E_2. Let's again determine this voltage using the voltage division method.

$$E_{2A} = \frac{R_1}{R_1 + R_2} \times E_2$$
$$= \frac{200}{100 + 200} \times -6$$
$$= -4\ Volt$$

The voltage of the A junction with respect to the ground is the sum of the two voltages that we found. Let's denote voltage as "E_p", and we can find:

$$E_p = E_{A1} + E_{A2}$$
$$= 10 + (-4)$$
$$= 6\ Volt$$

1.2 Thevenin Theorem

One of the most common methods used in the analysis of the electrical circuits is the Thevenin Theorem. Using this method, no matter how complicated the electrical circuit is, it can be converted into a single voltage source E_{Th} and a resistor R_{Th} which is connected serially to E_{Th}.

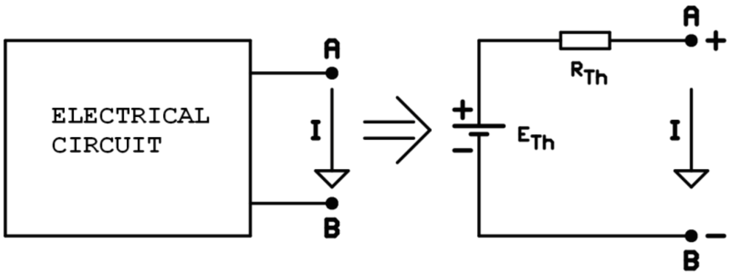

Figure 5: Block diagram of Thevenin conversion.

The E_{Th} voltage in Figure 5 is the open circuit voltage of the A and B points. R_{Th} is the equivalent resistance between A and B points when the source is removed. This simple circuit illustration is called the "Thevenin equivalent circuit". The circuit analysis can easily be done after the equivalent circuit is constructed.

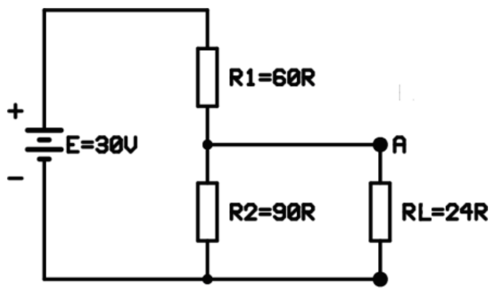

Figure 6: Simple example is given in order to find Thevenin equivalent circuit.

Determine the voltage across the load resistor in Figure 6 by sing the Thevenin Theorem. The voltage across the terminals of the R_2 resistor is the open circuit voltage of A and B points. At the same time, this is the Thevenin voltage as following equation, $E_{R2} = E_{AB} = E_{Th}$.

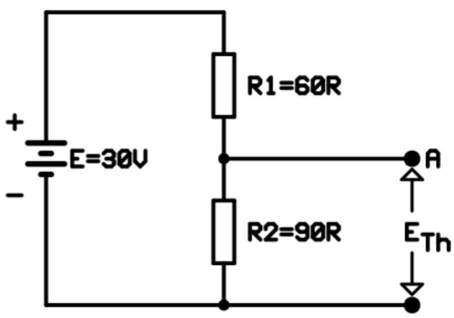

Figure 7: Thevenin voltage is determined across the load resistor.

Let's calculate this voltage using voltage division rule.

$$\begin{aligned} E_{Th} &= \frac{R_2}{R_1 + R_2} \times E \\ &= \frac{90}{60 + 90} \times 30 \\ &= 18\ Volt. \end{aligned}$$

In order to find the Thevenin resistance (R_{Th}) of the circuit, the source is removed and its terminals are short-circuited. At this point, the resistance between A and B points is the Thevenin Resistance, R_{Th}.

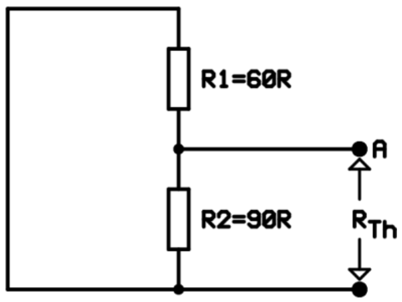

Figure 8: Thevenin resistor is determined across the load resistor.

In Figure 8, the R_1 and the R_2 resistors are parallel to each other. The equivalent resistance of the parallel two resistors can most easily be found by multiplying two values and then dividing the result to the sum of the resistances.

$$\begin{aligned} R_{Th} &= \frac{R_1 \times R_2}{R_1 + R_2} \\ &= \frac{60 \times 90}{60 + 90} \\ &= 36\ \Omega \end{aligned}$$

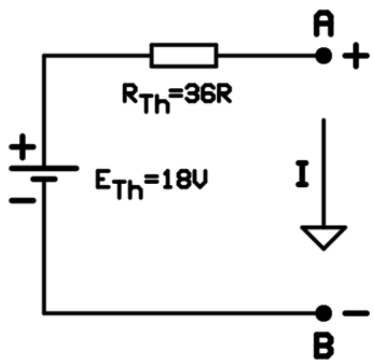

Figure 9: Equivalent Thevenin circuit of example circuit.

The Thevenin Equivalent of a circuit can be seen in Figure 9. Until this time, the load has not yet been used in the analysis. At this point, the load is connected to the Thevenin equivalent circuit in order to measure the voltage on the load, E_L, and the current passing through it.

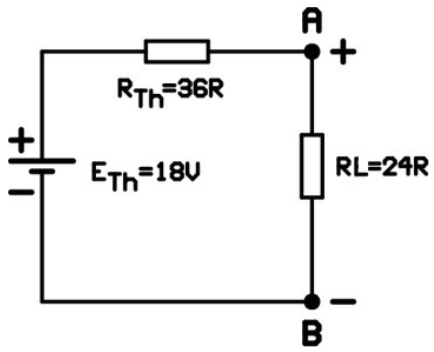

Figure 10: Equivalent Thevenin circuit of example circuit with load resistor.

In Figure 10, the R_{Th} and the R_L resistors are in series. Therefore, circuit current flows through the load.

$$\begin{aligned} I_L &= \frac{E_{Th}}{R_L + R_{Th}} \\ &= \frac{18}{36+24} \\ &= 0,3\ A \end{aligned}$$

The voltage between the terminals of the load is determined with the voltage division method.

$$\begin{aligned} E_L &= \frac{R_L}{R_L + R_{Th}} \times E_{Th} \\ &= \frac{24}{24+36} \times 18 \\ &= 7,2\ Volt \end{aligned}$$

Also, voltage of load can be found easily with the Ohm's law.

$$\begin{aligned} E_L &= I_L \times R_L \\ &= 0,3 \times 24 \\ &= 7,2\ Volt \end{aligned}$$

1.3 Norton Theorem

Another common method used in the analysis of the electrical circuits is the Norton Theorem. Using this method, no matter how complicated the electrical circuit is, it can be converted to a single current source, I_N, and a resistor, R_N, connected in parallel to the source.

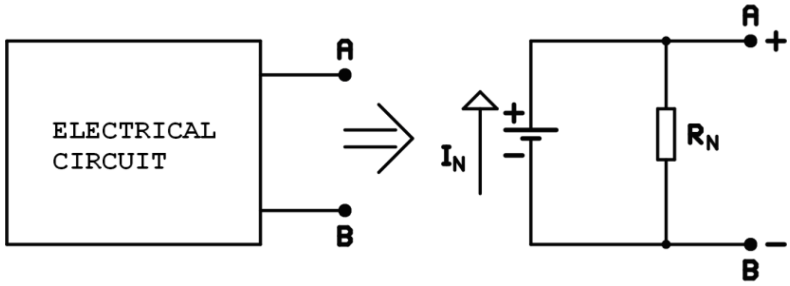

Figure 11: Block diagram of Norton conversion.

I_N is the current passing when the A and B points are short-circuited. R_N is the resistance when looking through the A and B points. The representation of the electrical circuit in this manner is called the "Norton Equivalent Circuit". After this simplification, the solution to the problem can be found easily. Let's determine the current passing through the load in Figure 12 using the Norton Theorem.

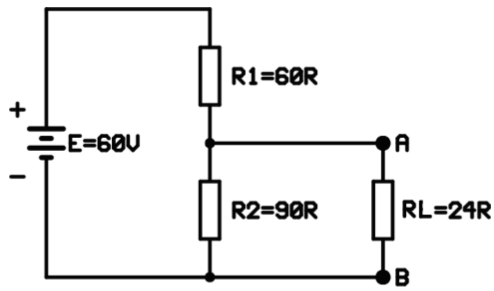

Figure 12: Simple example in order to find Norton equivalent circuit.

In order to convert the circuit to the Norton Equivalent Circuit, the load is removed from the circuit and its terminals are short-circuited. Then, the Norton current I_N is found.

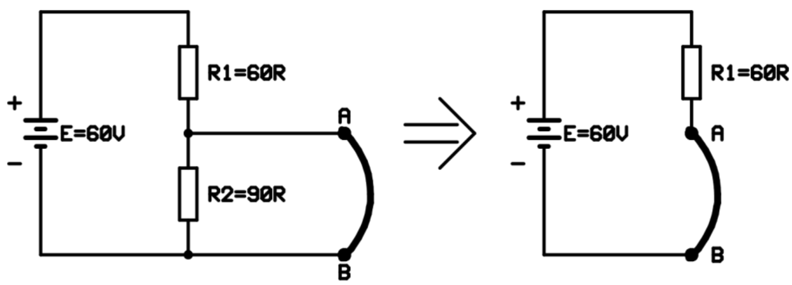

Figure 13: Load resistance is short-circuited in order to find Norton current.

When the A and B points are short-circuited, the R_2 resistor has no effect. This situation can be seen in Figure 13. The Norton current I_N is found using Ohm's law.

$$\begin{aligned} I_N &= \frac{E}{R_1} \\ &= \frac{60}{60} \\ &= 1\ A \end{aligned}$$

The source is removed from the circuit in order to find the Norton Resistance, R_N. After shorting the source, the resistance between A and B points is calculated.

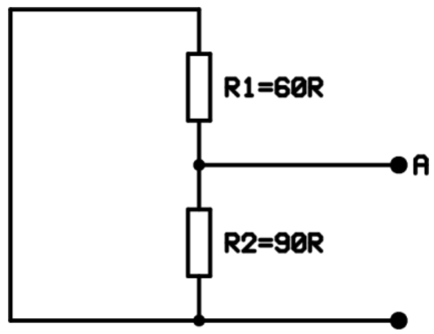

Figure 14: The source and load is removed in order to find Norton resistance.

In Figure 14, R_1 and R_2 are parallel. The overall resistance of the circuit is R_N.

$$\begin{aligned} R_N &= \frac{R_1 \times R_2}{R_1 + R_2} \\ &= \frac{60 \times 90}{60 + 90} \\ &= 36\ \Omega \end{aligned}$$

After finding I_N and the R_N, Norton equivalent of the circuit can be drawn.

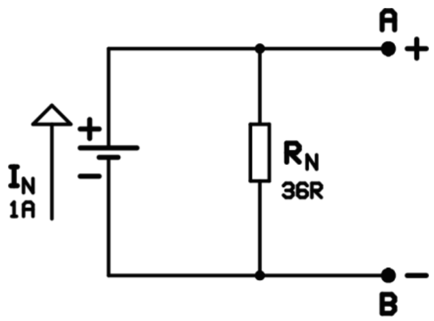

Figure 15: Norton equivalent circuit of example circuit.

By connecting load to the Norton equivalent circuit, the solution is completed.

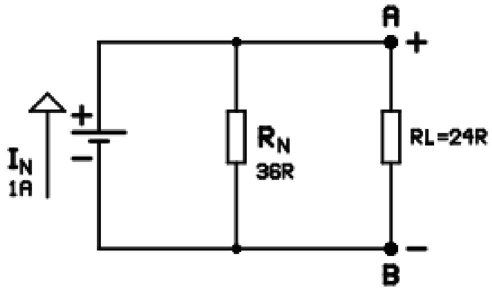

Figure 16: Norton equivalent circuit of example circuit with load.

The branch currents in Figure 16 are calculated using current division method. The current division method is a practical method that is also derived from Ohm's law and Kirchhoff's laws. This method resembles the voltage division method.

$$I_L = \frac{R_N}{R_L + R_N} \times I \ and \ I_N \ = \frac{R_L}{R_L + R_N} \times I$$

Equations of voltage division and current division methods are different from each other because the current and the resistance values are inversely proportional.

$$\begin{aligned} I_L &= \frac{R_N}{R_L + R_N} \times 1 \\ &= \frac{36}{24 + 36} \times 1 \\ &= 0,6 \ A \\ I_N &= \frac{R_L}{R_L + R_N} \times 1 \\ &= \frac{24}{24 + 36} \times 1 \\ &= 0,4 \ A \end{aligned}$$

The voltage between the ends of the load is found using Ohm's law.

$$\begin{aligned} E_L &= I_L \times R_L \\ &= 0,6 \times 24 \\ &= 14,4 \ Volt \end{aligned}$$